Effects of anisotropic parameters on the acoustical behavior of porous materials with Biot's model through the finite element method

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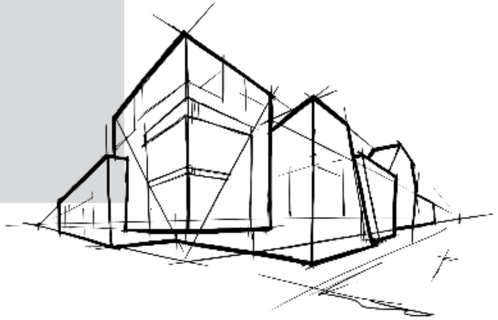
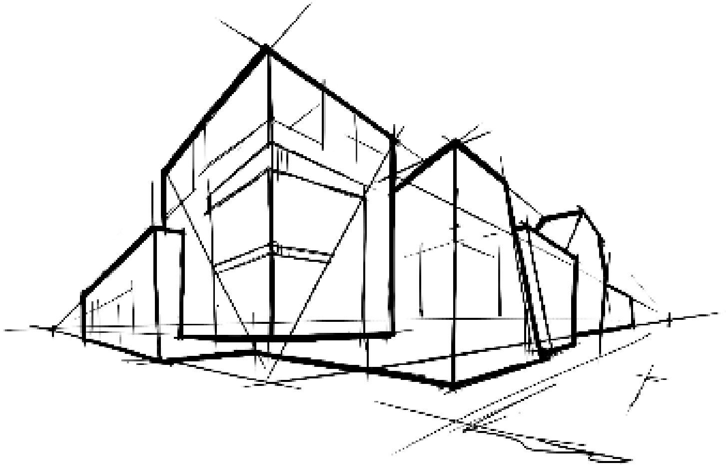
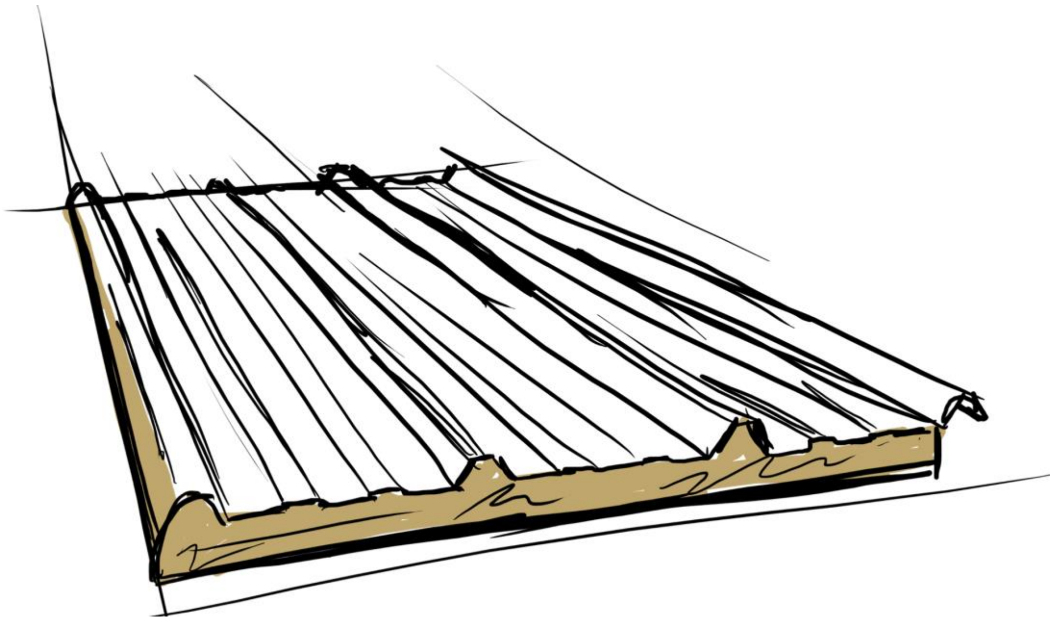


Motivation

- Building acoustics
- **Fibrous materials →** stone wool
- Sound **absorption** and **insulation** properties
- It's **anisotropic parameters** have not been fully researched yet
- **Anisotropy is key**, since it is not a transversely isotropic material
- Overall **knowledge gap** regarding fibrous materials
- In **building elements**, a better understanding of it will be beneficial for **optimization**





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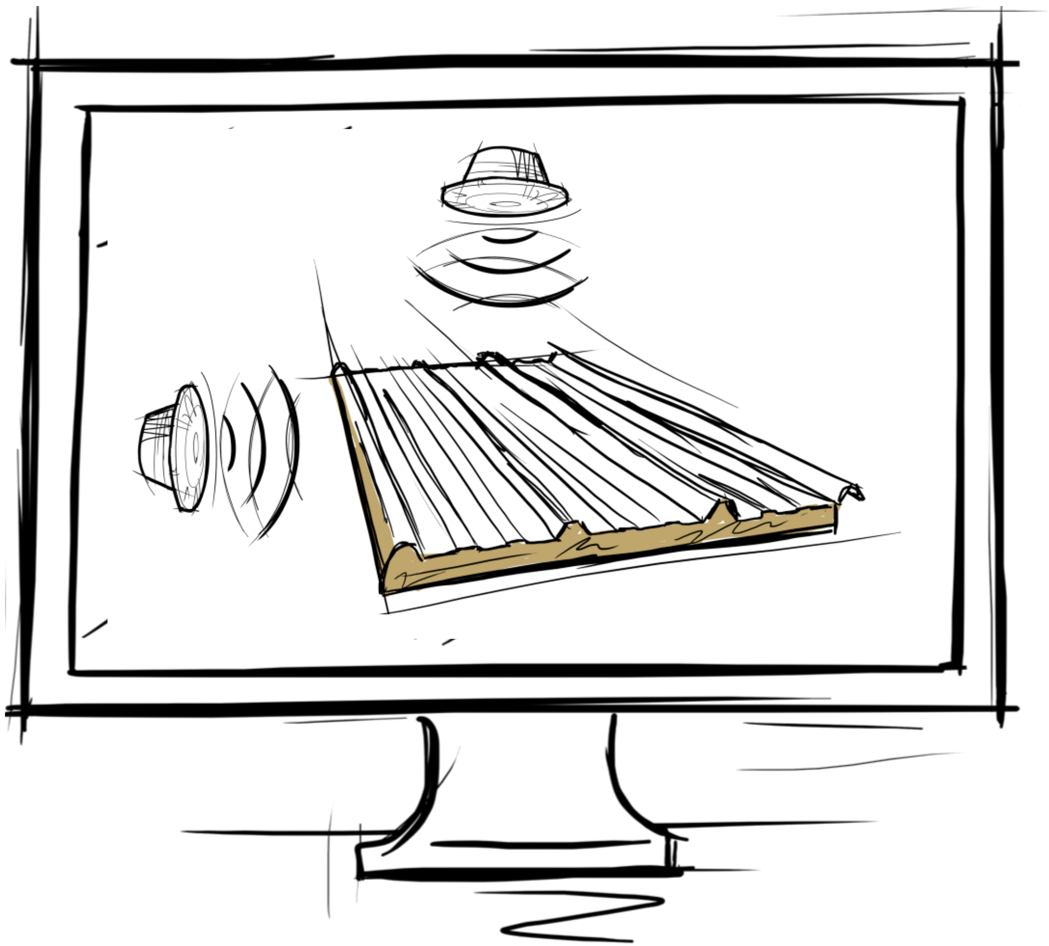
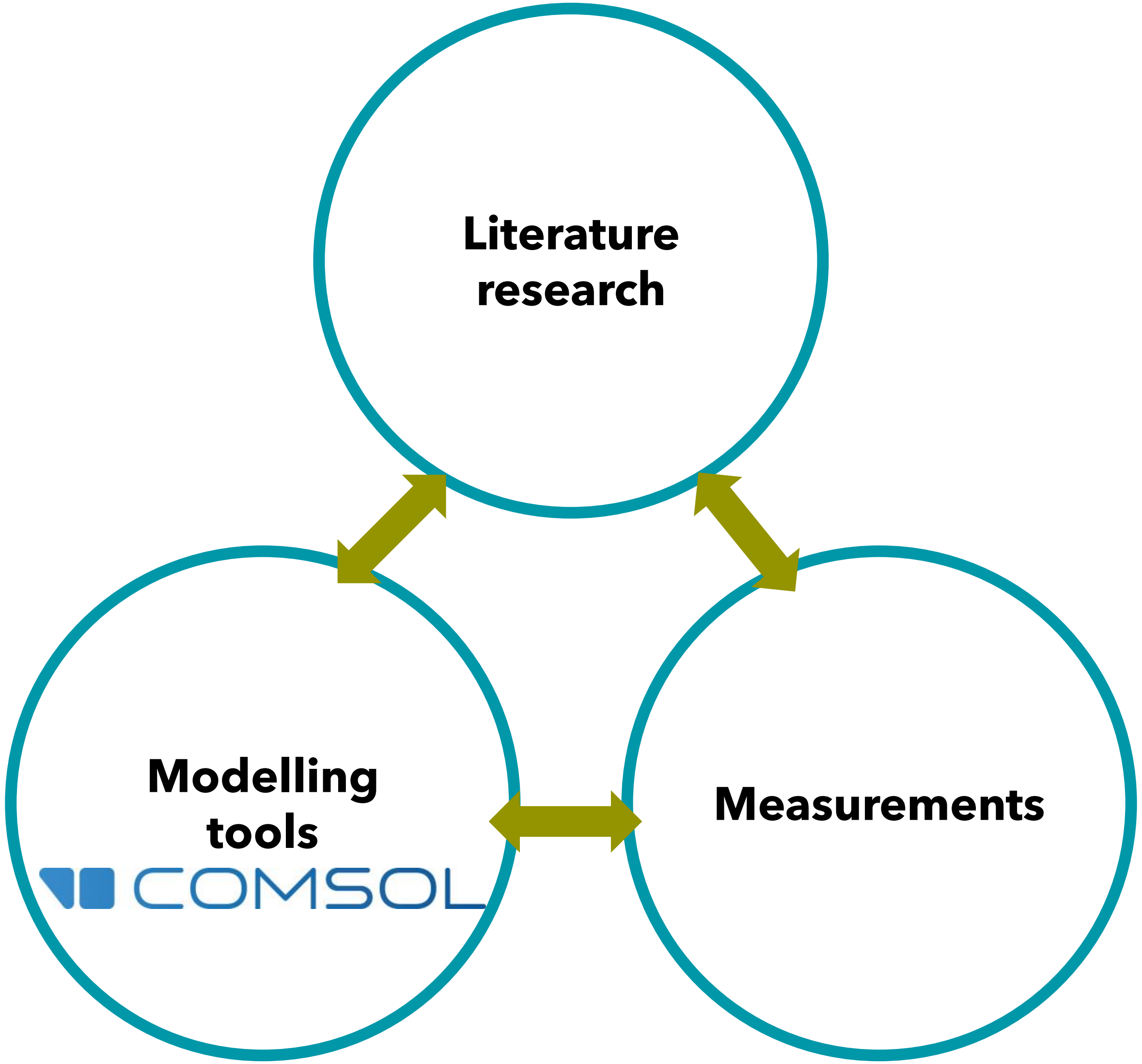
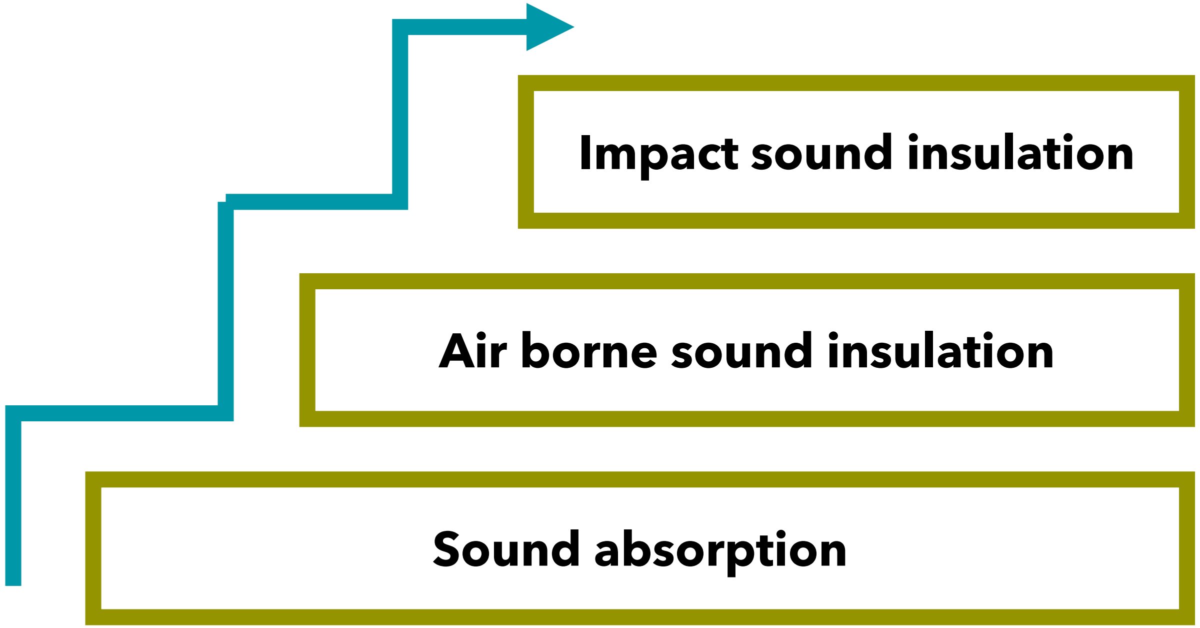
How to accurately **simulate** the **acoustic behavior** of a porous **fibrous material** as part of a building element, and to what extent is its **anisotropy** relevant?



Research project





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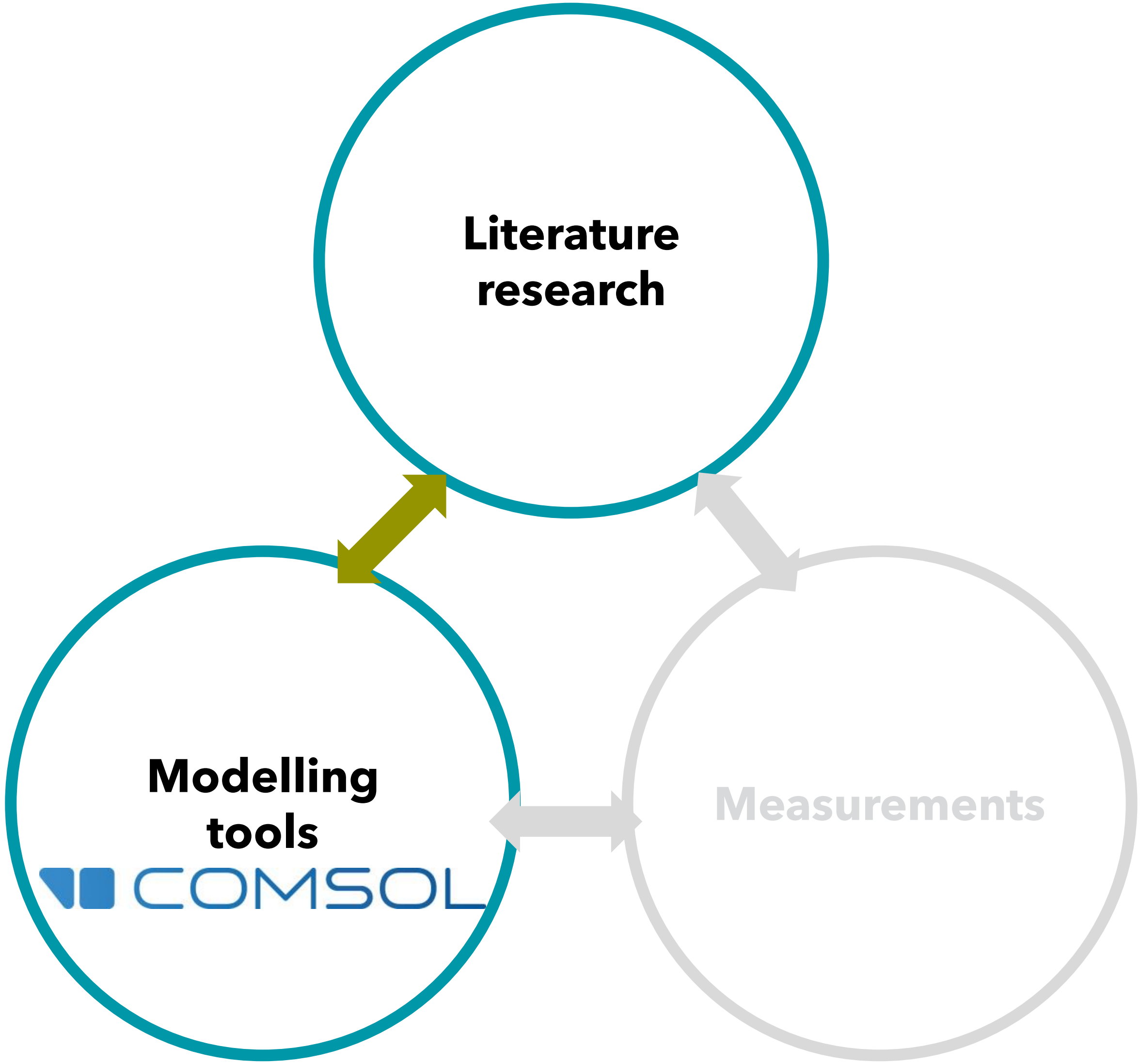

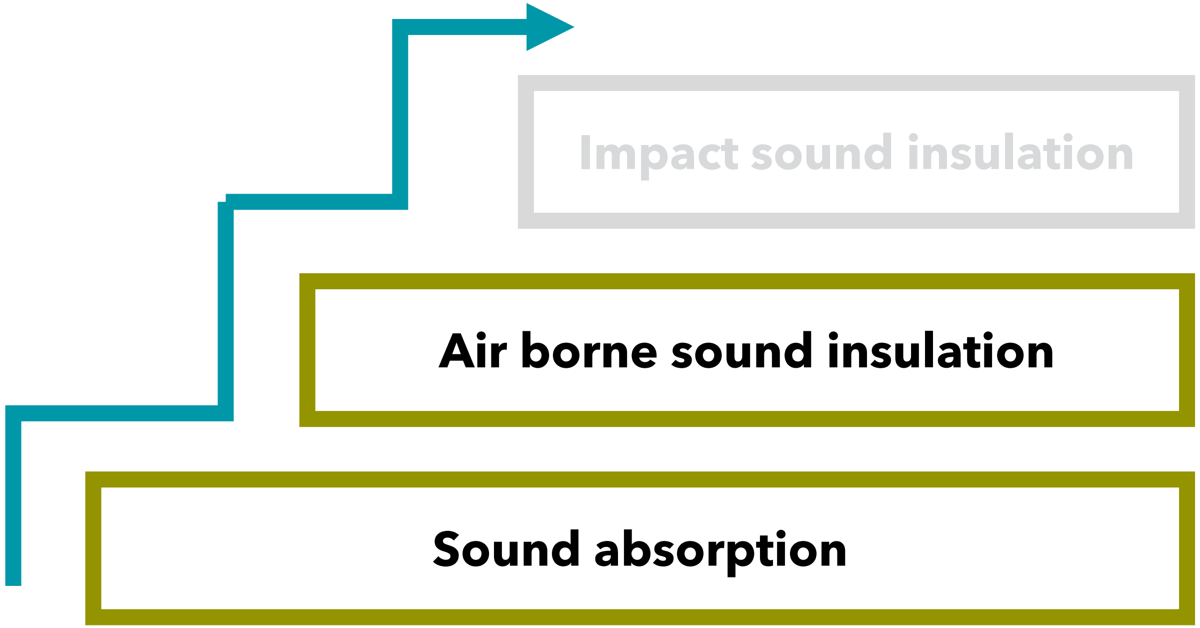

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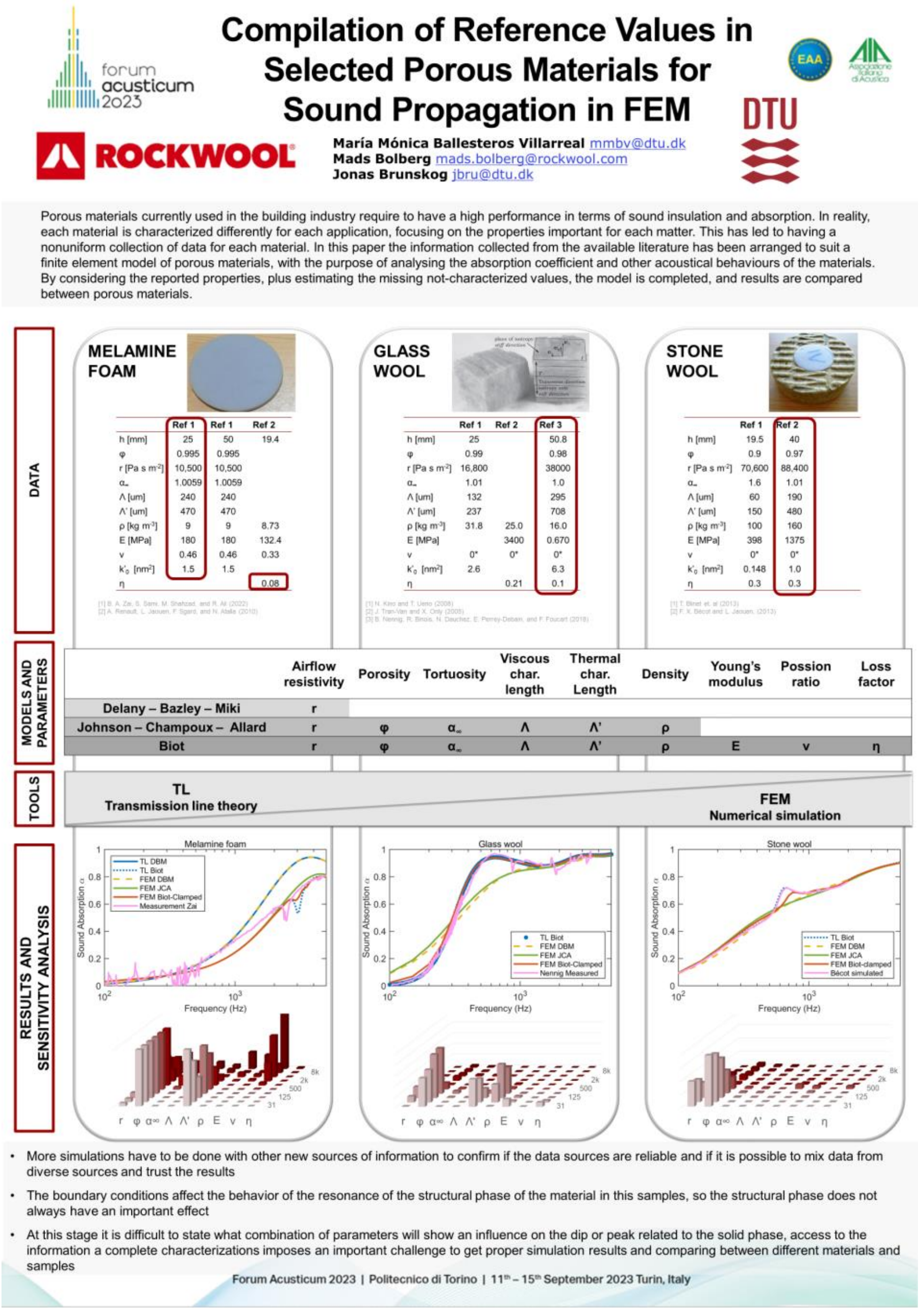
Previous work

- Data compilation work
  - Data
    - Melamine foam
    - Glass wool
    - Stone wool
  - Models
    - DBM
    - JCA
    - Biot

- **Lack of available information** with all the necessary **parameters for the Biot model**



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How can porous materials be studied?

Delany-Bazley-Miki

Johnson-Champoux-Allard (JCA)

Biot

r Airflow resistivity

ϕ Porosity
α_∞ Tortuosity
Λ Viscous char. length
Λ' Thermal char. Length
ρ Density

E Young's modulus
ν Poisson ratio
η Loss factor

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Johnson-Champoux-Allard model (JCA)

Bulk modulus

$$K_{eq} = \frac{\gamma P_0/\phi}{\gamma - (\gamma - 1)\left[1 - j\frac{8\kappa}{\Lambda'^2 C_p \rho_0 \omega}\sqrt{1 + j\frac{\Lambda'^2 C_p \rho_0 \omega}{16\kappa}}\right]^{-1}}$$

Equivalent density

$$\rho = \frac{\rho_0 \alpha_\infty}{\phi}\left[1 + \frac{r\phi}{j\omega\rho_0\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2 \eta_v \rho_0 \omega}{r^2 \Lambda^2 \phi^2}}\right]$$

Anisotropic

$$\rho_{eq} = \begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix}$$

Airflow resistivity can be anisotropic

Biot model

$$-\rho_{av}\omega^2\mathbf{u} + \rho_f\omega^2\mathbf{w} - \nabla\cdot\sigma = 0$$
$$-\rho_f\omega^2\mathbf{u} - \omega^2\rho_c(\omega)\mathbf{w} + \nabla p_f = 0$$

$$\rho_c(\omega) = \frac{\alpha_\infty}{\phi}\rho_f + \frac{\mu(\omega)}{i\omega\kappa}$$

$$\mu(\omega) = \mu\sqrt{(1 + \frac{4i\omega\alpha_\infty^2\mu\rho_f}{r^2\Lambda^2\phi^2})}$$

Airflow resistivity can be anisotropic

Stress tensor – Hooke's law

$$\begin{cases}\hat\sigma_{xx} = Fe_{zz} + Fe_{yy} + Ce_{xx}\\ \hat\sigma_{yy} = Ae_{zz} + (2G + A)e_{yy} + Fe_{xx}\\ \hat\sigma_{zz} = (2G + A)e_{zz} + Ae_{yy} + Fe_{xx}\\ \hat\sigma_{yx} = 2G'e_{yx}\\ \hat\sigma_{xz} = 2G'e_{xz}\\ \hat\sigma_{zy} = 2Ge_{zy},\end{cases}$$

$$A = \frac{E(E'\nu + E\nu'^2)}{(1+\nu)(E' - E'\nu - 2E\nu'^2)}$$
$$F = \frac{EE'\nu'}{E' - E'\nu - eE\nu'^2}$$
$$C = \frac{E'^2(1-\nu)}{E' - E'\nu - 2E\nu'^2}$$
$$G = \frac{E}{2(1+\nu)}.$$

Transversally isotropic Young's modulus

COMSOL version 6.1

Transversally isotropic glass wool

Nennig, Binois, Dauchez, et. al. 2018

h [mm]	50.8
ϕ	0.98
rt [Pa s m^{-2}]	38,000
ri[Pa s m^{-2}]	16,720
α_∞	1.0
Λ [um]	295
Λ′ [um]	708
ρ [kg m^{-3}]	16.0
Et [MPa]	0.670
Ei [MPa]	142
ν	0*
k'_0 [nm^2]	6.3
η	0.1

JCA

Anisotropic airflow resistivity

$$\rho = \frac{\rho_0 \alpha_\infty}{\phi}\left[1 + \frac{r\phi}{j\omega\rho_0\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty^2\eta_v\rho_0\omega}{r^2\Lambda^2\phi^2}}\right]$$

Biot (structural)

Anisotropic Young's modulus

$$-\rho_{av}\omega^2\mathbf{u} + \rho_f\omega^2\mathbf{w} - \nabla\cdot\sigma = 0$$

$$-\rho_f\omega^2\mathbf{u} - \omega^2\rho_c(\omega)\mathbf{w} + \nabla p_f = 0$$

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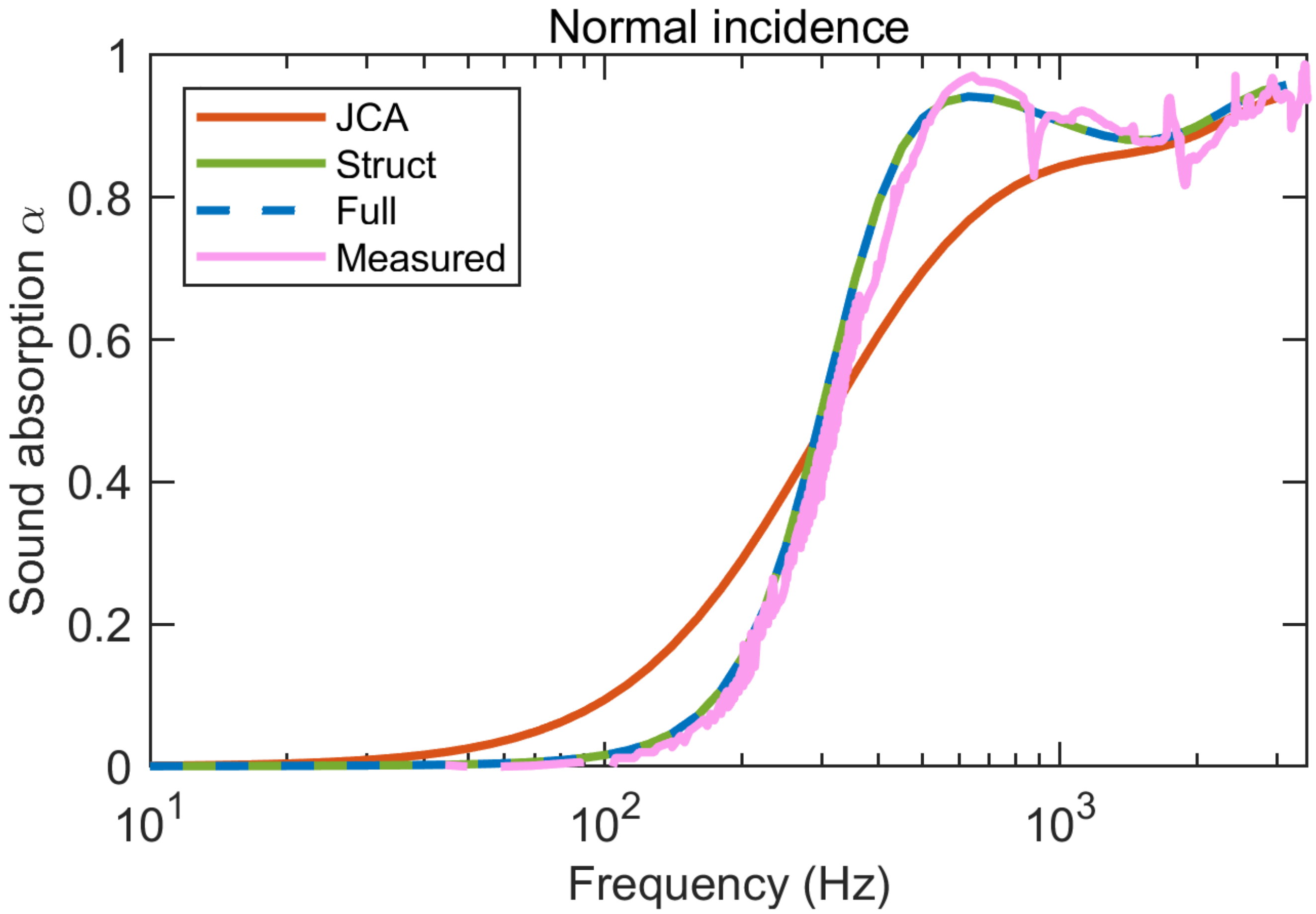
Nennig, Binois, Dauchez, et. al. 2018

- Sound absorption
- Infinite sample
  - For oblique incidence
- Previous work included only JCA and Biot-Structural models vs measured results
- **Current work: Full Biot model, available in COMSOL version 6.2**

JCA – Only anisotropic airflow resistivity **r**

Struct- Biot model with only anisotropic elastic parameters **E, G, ν**

Full- Biot model with anisotropic airflow resistivity **r** and elastic parameters **E, G, ν**



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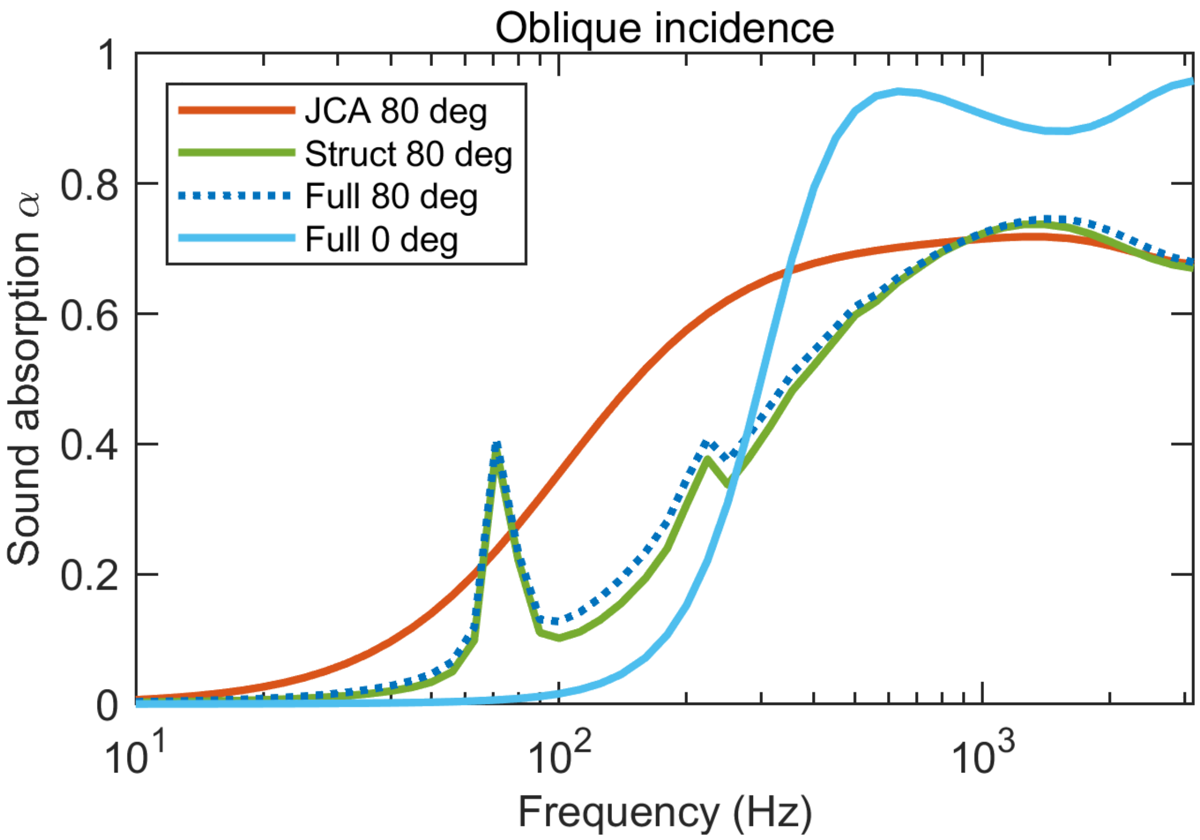
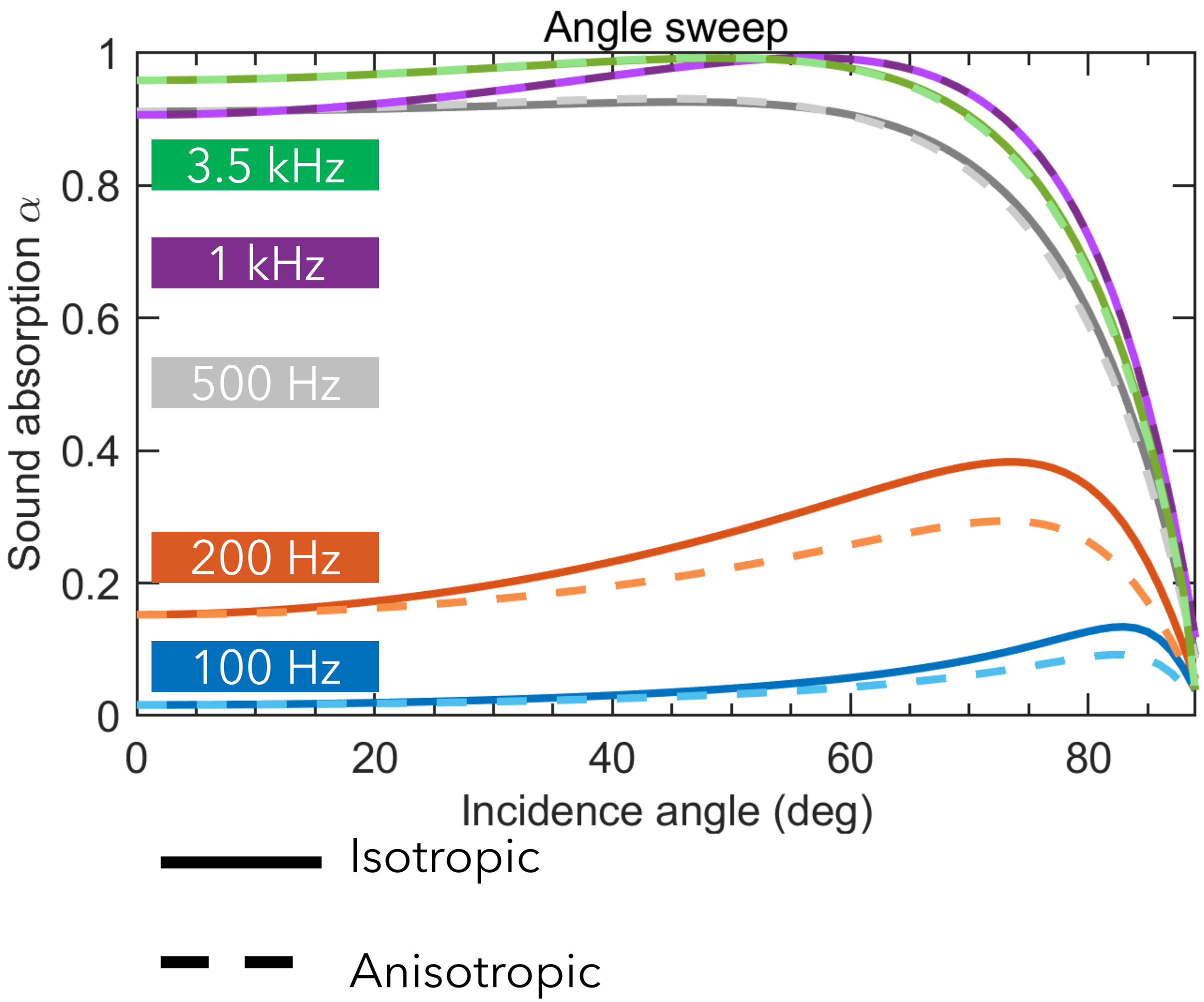
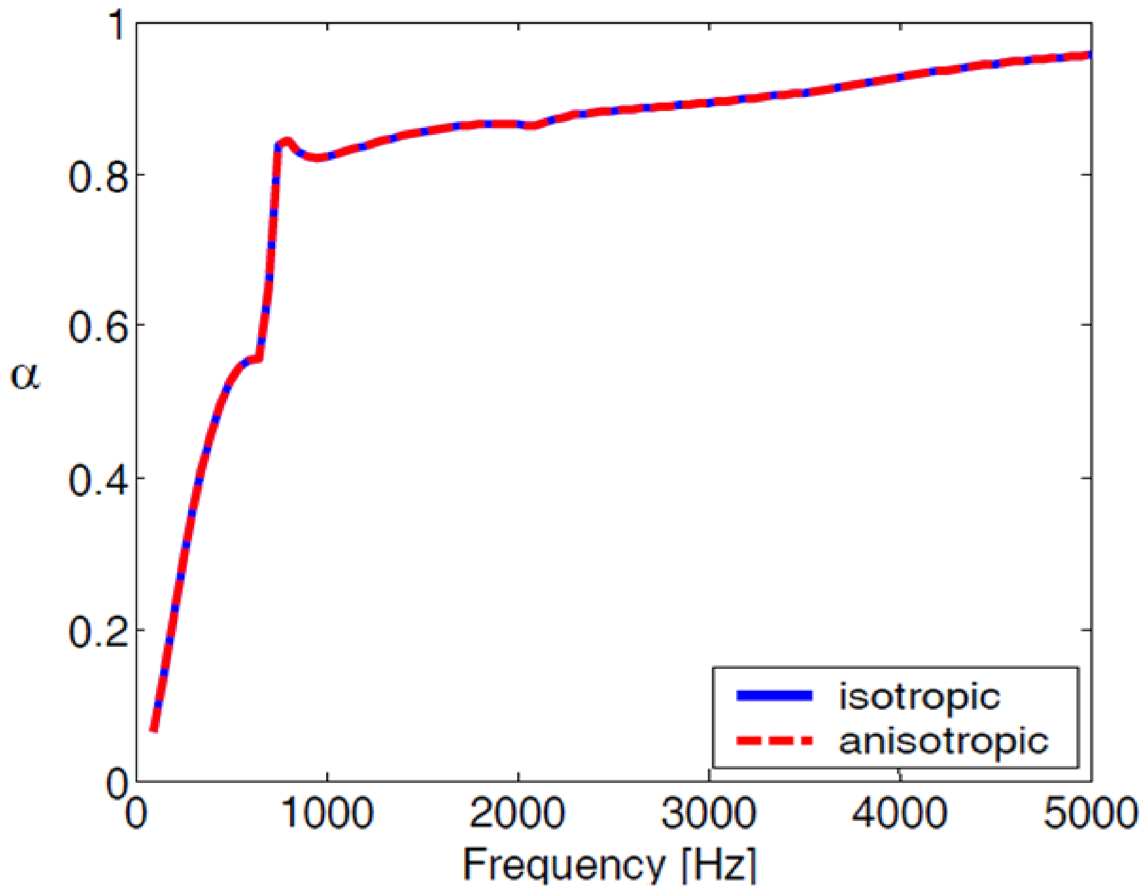
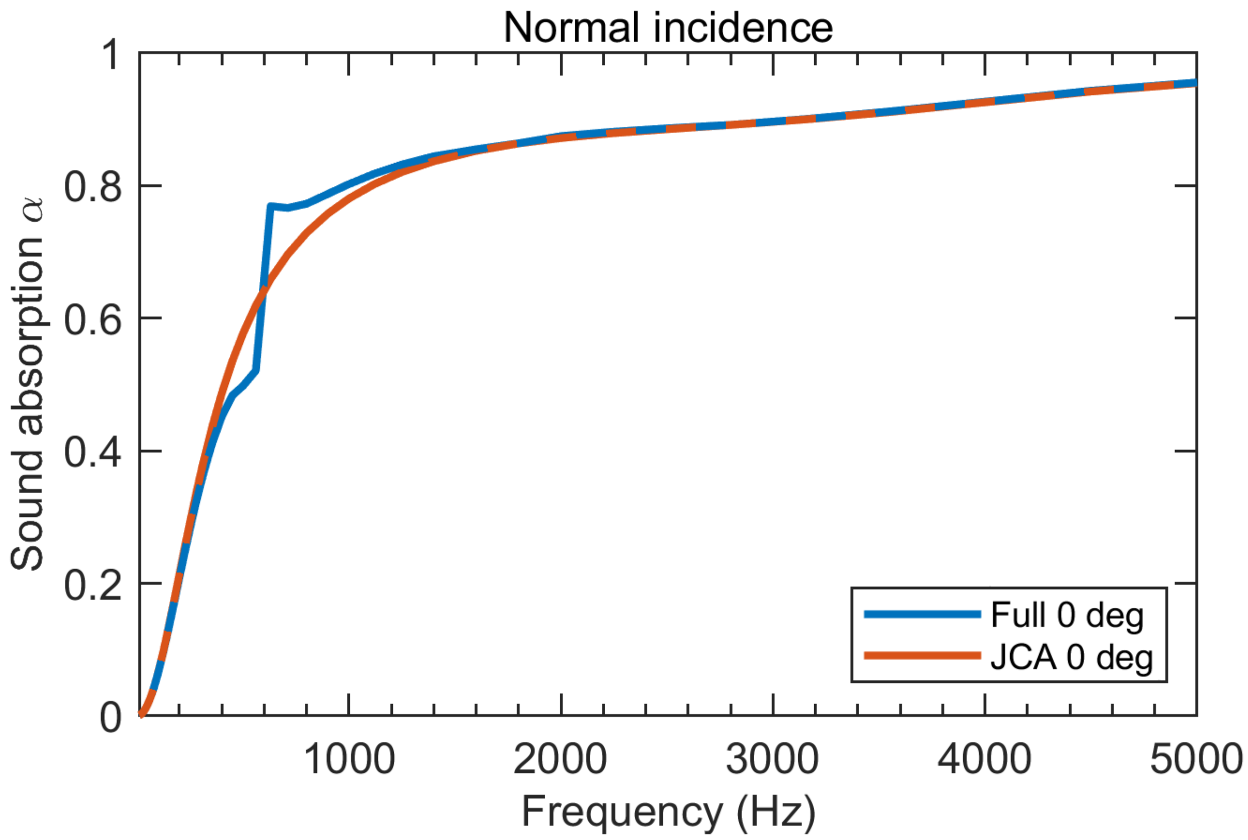




Geebelen, 2008:
Stonewool infinite sample
Normal incidence





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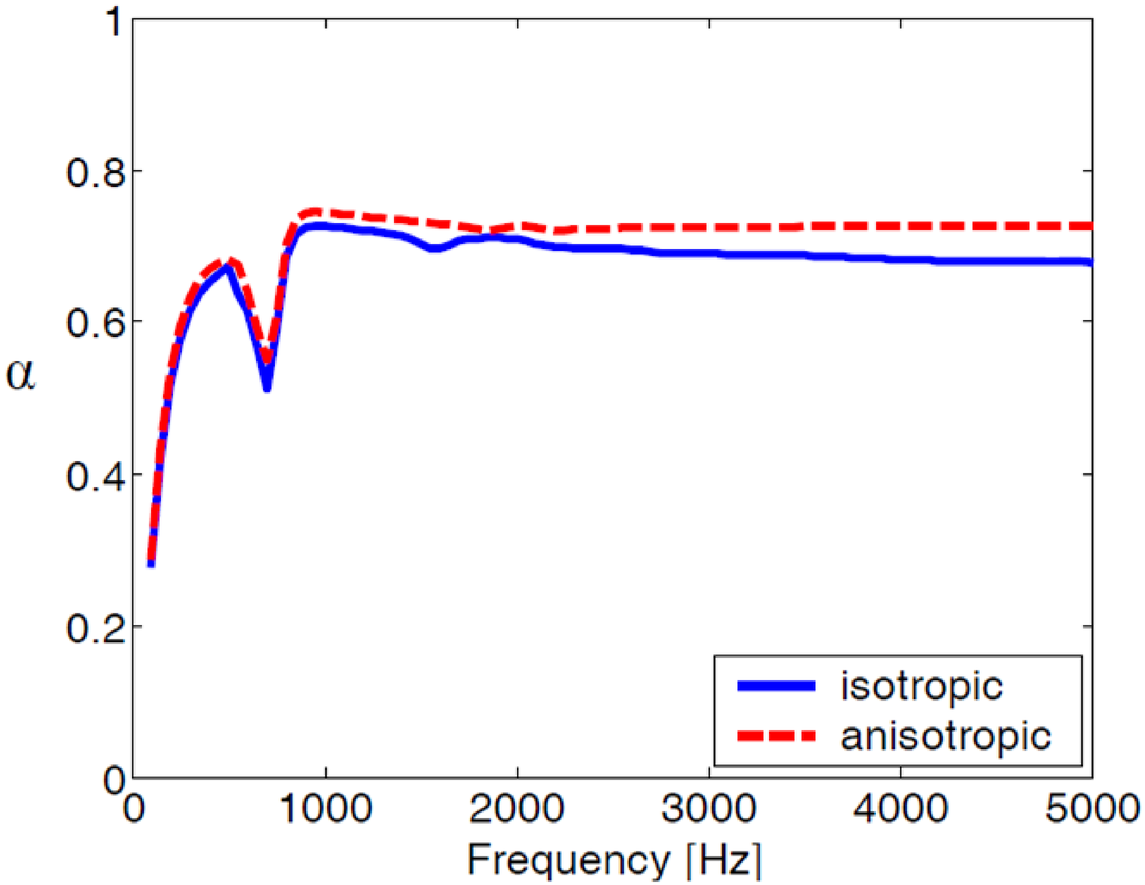
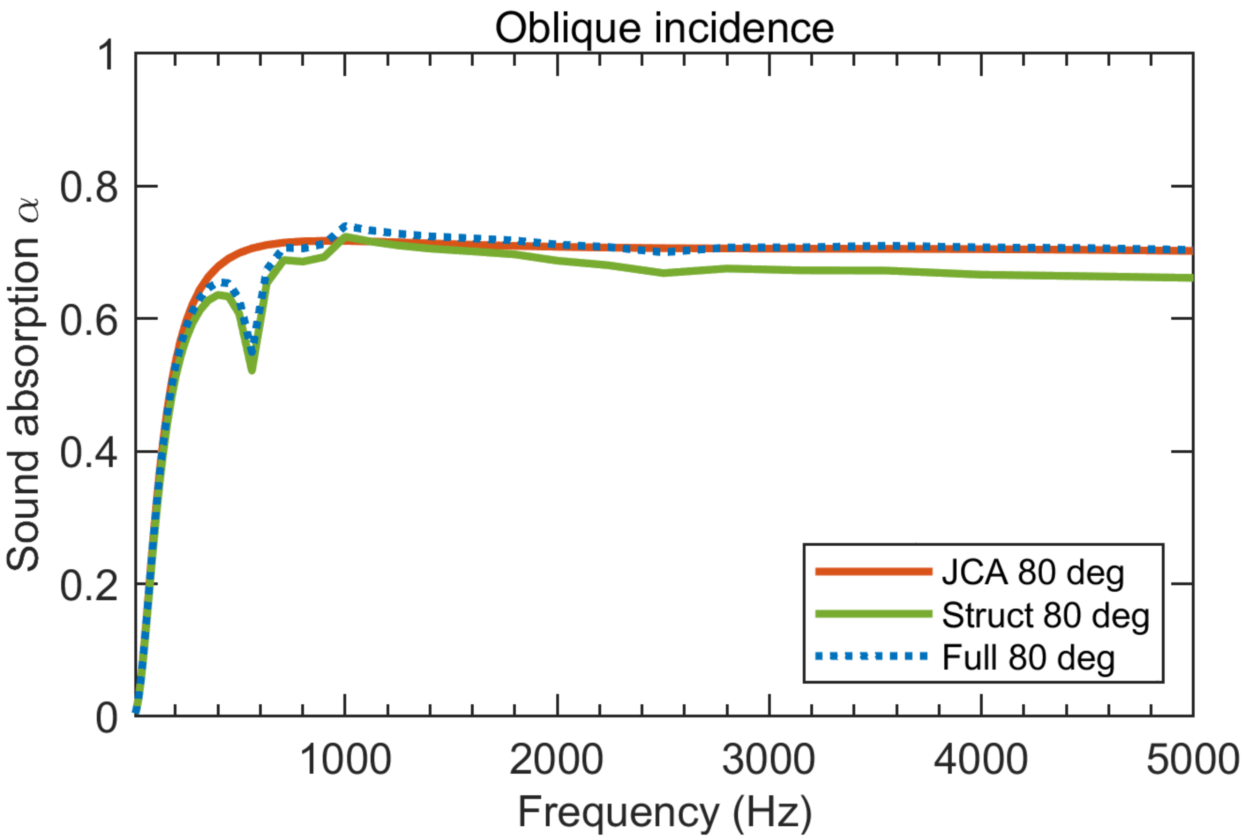
Geebelen, 2008:
Stonewool infinite sample
Oblique incidence 80°





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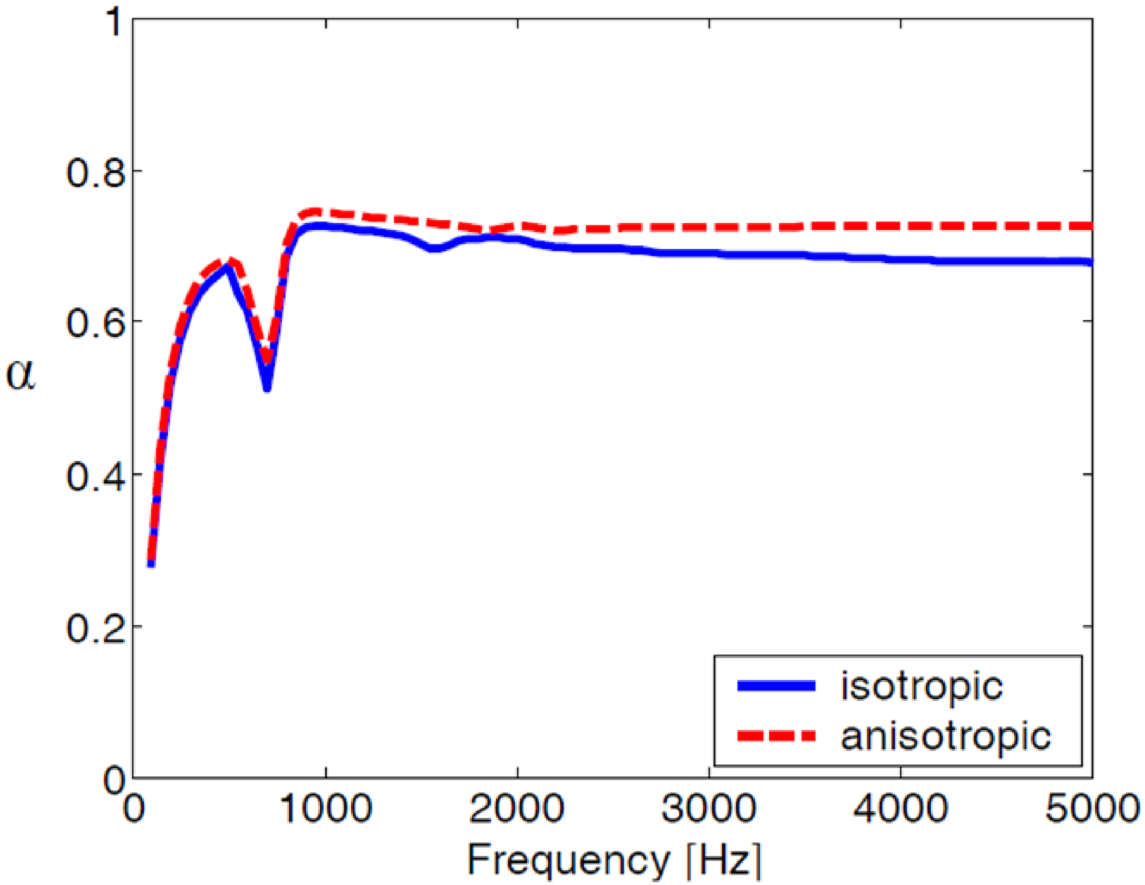

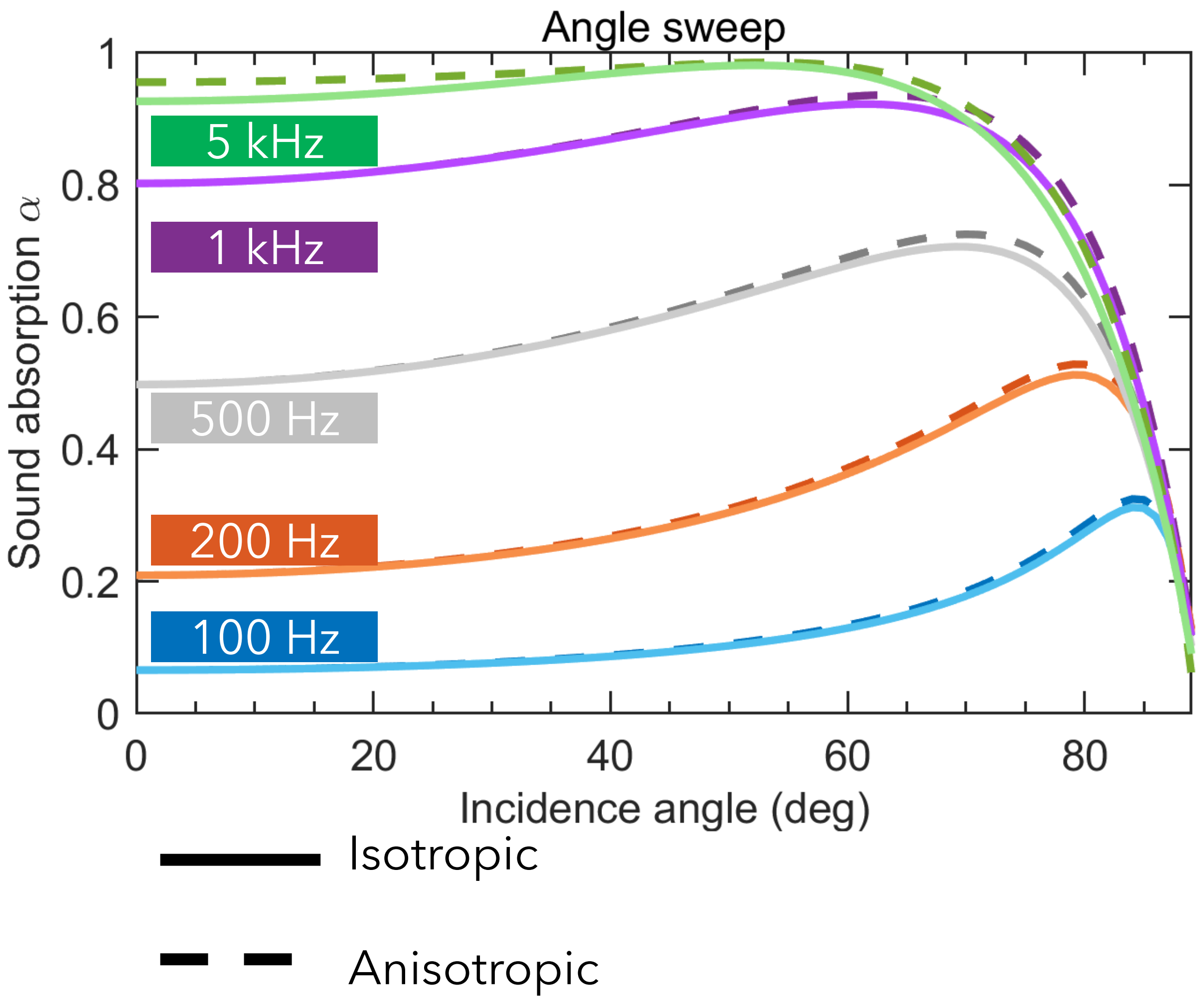

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Work in progress: Sandwich panels (isotropic), Panneton, 1996

- Sound Transmission Loss
- Finite sample
- Isotropic core
- Biot model
- **Future work: fill with anisotropic core**

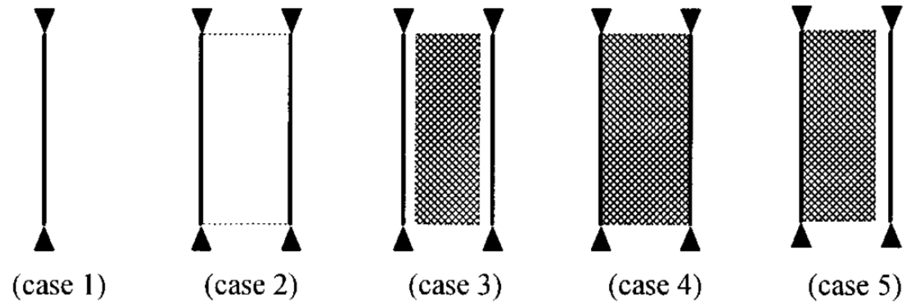
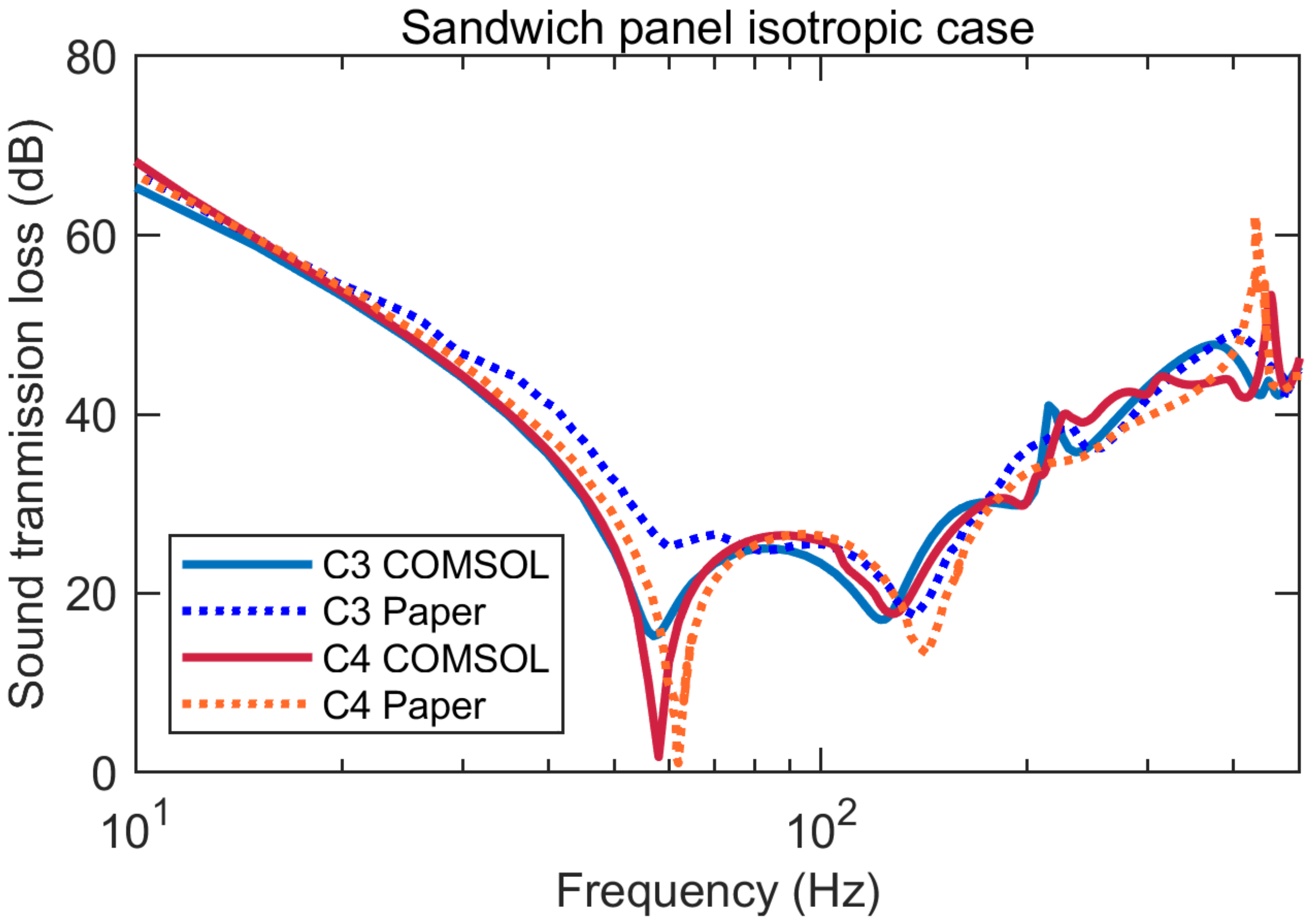


FIG. 2. Studied design configurations: (case 1) single plate; (case 2) unlined; (case 3) unbonded layer; (case 4) bonded layer; (case 5) bonded–unbonded layer.

Panneton, 1996



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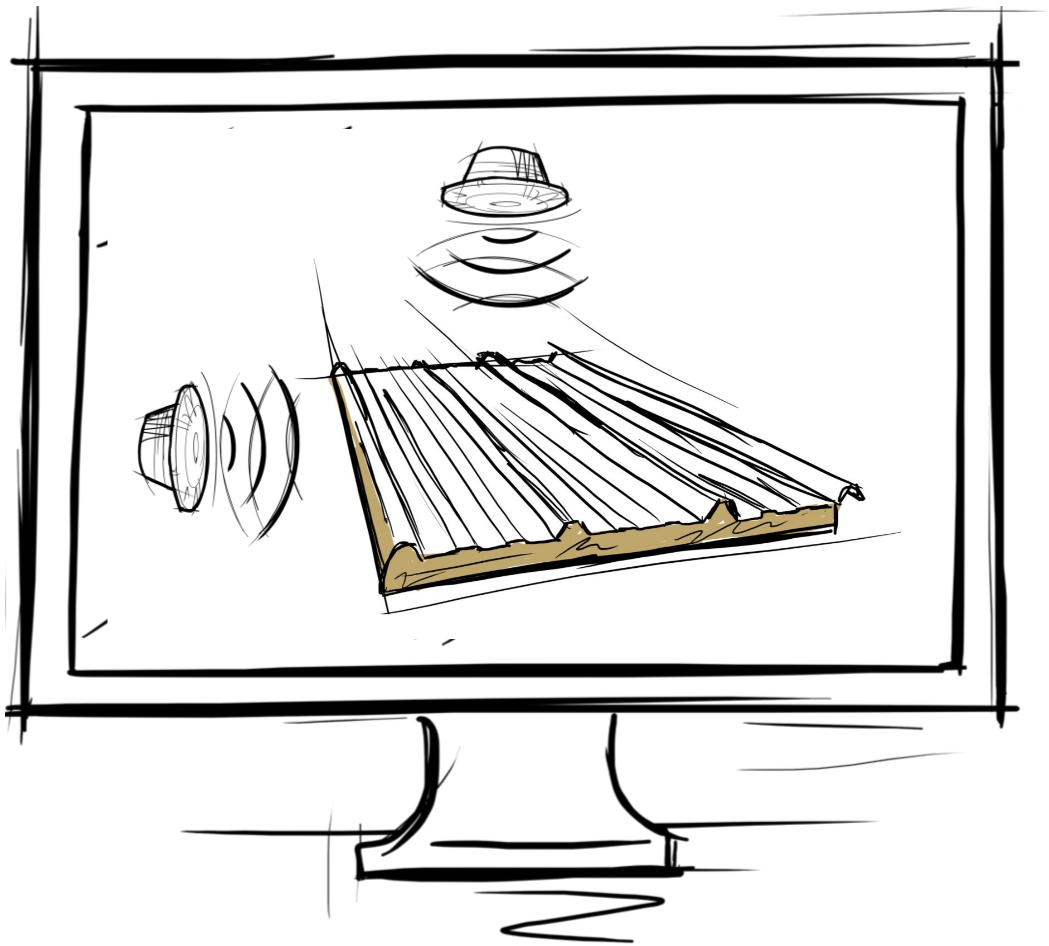
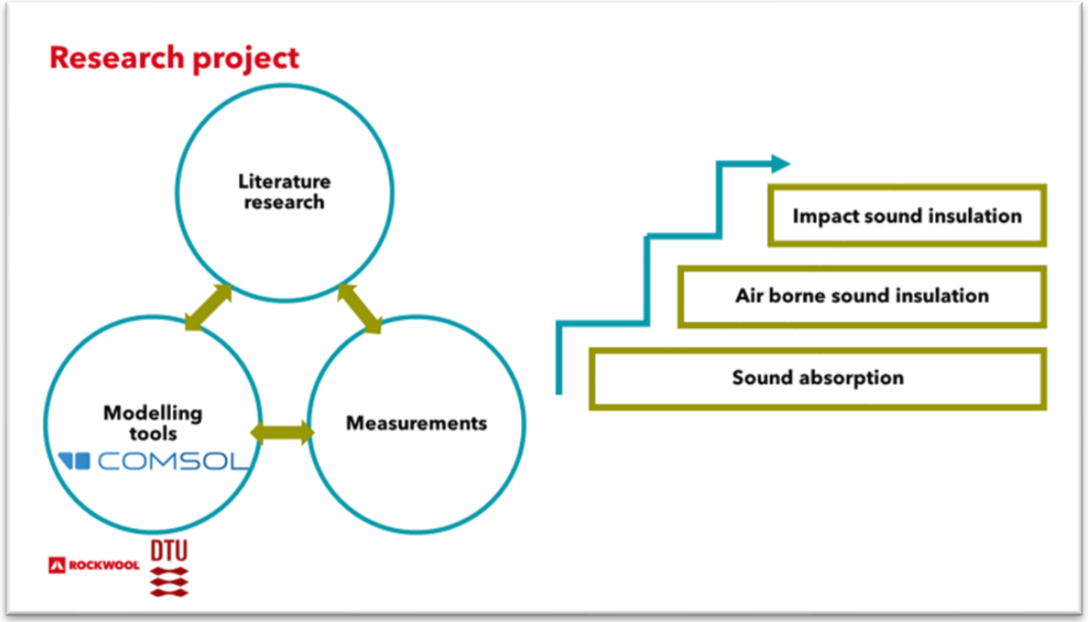


Conclusions & future work

- Anisotropy effects in both models have the same direction and are similar in size. They ~~might~~ add up in a **fully anisotropic model**.
- Oblique incidence studies offer more information regarding the anisotropy in both the airflow resistivity and elastic parameters

➢Characterization for anisotropic Biot parameters
   ➢Gather more information from literature if available
➢Simulation of sandwich panels and sound insulation
➢Simulation of floors and impact sound



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