

Statistical characterisation of porous media from sound absorption and ultrasound transmission

Jacques Cuenca¹, Tomás S. Gómez Méndez², Naima Sebaa³, David Jun^{4,5}, Laurent De Ryck¹, Christ Glorieux²

¹ Siemens Industry Software, Leuven, Belgium

² KU Leuven, Department of Physics and Astronomy, Laboratory for Acoustics – Soft Matter and Biophysics, Heverlee, Belgium

³ École Nationale Supérieure des Technologies Avancées, Algiers, Algeria

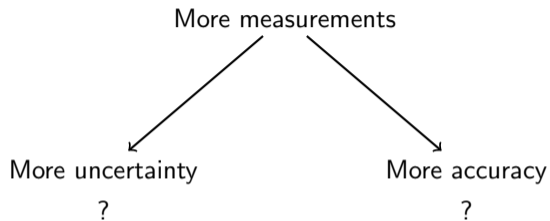
⁴ Brno University of Technology, Faculty of Civil Engineering, Brno, Czech Republic

⁵ KU Leuven, Department of Architecture, Campus Brussels and Ghent, Belgium

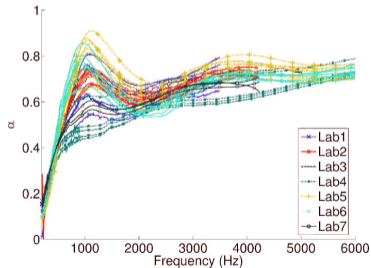
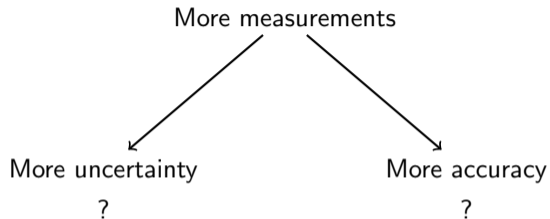
SIEMENS



Question

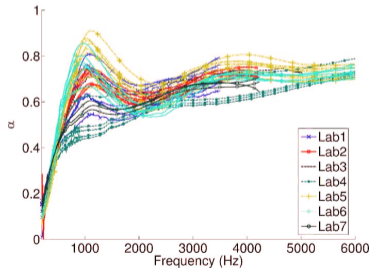
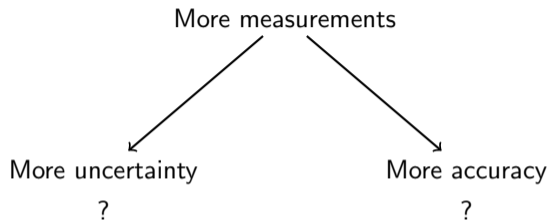


Question

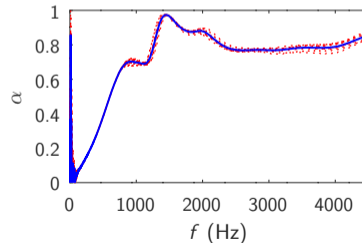


Round robin tests, Horoshenkov et al. JASA 2007

Question




Round robin tests, Horoshenkov et al. JASA 2007



Repeatability test (1 material, 1 impedance tube)

Previous work




ELSEVIER

Contents lists available at [ScienceDirect](https://www.sciencedirect.com)

Mechanical Systems and Signal Processing


journal homepage: www.elsevier.com/locate/ymssp



Deterministic and statistical methods for the characterisation of poroelastic media from multi-observation sound absorption measurements

J. Cuenca ^a, P. Göransson ^{b,*}, L. De Ryck ^a, T. Lähivaara ^c

^a *Siemens Industry Software, Interleuvenlaan 68, BE-3001 Leuven, Belgium*
^b *Department of Aeronautical and Vehicle Engineering, KTH Royal Institute of Technology, Teknikringen 8, SE-10044 Stockholm, Sweden*
^c *Department of Applied Physics, University of Eastern Finland, P.O. Box 1627, FIN-70211 Kuopio, Finland*



ARTICLE INFO

Communicated by Annie Ross

Keywords:
Poroelastic media
Parameter estimation
Coupled problems

ABSTRACT

This paper proposes a framework for the estimation of the transport and elastic properties of open-cell poroelastic media based on sound absorption measurements. The sought properties are the Biot-Johnson-Champoux-Allard model parameters, namely five transport parameters, two elastic properties and the mass density, as well as the sample thickness. The methodology relies on a multi-observation approach, consisting in combining multiple independent measurements

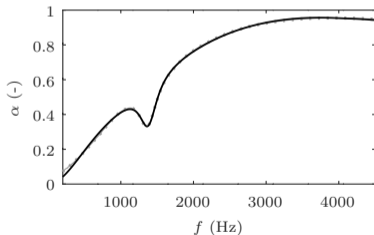
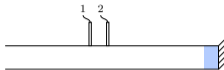
2/13

J. Cuenca (jacques.cuenca@siemens.com)

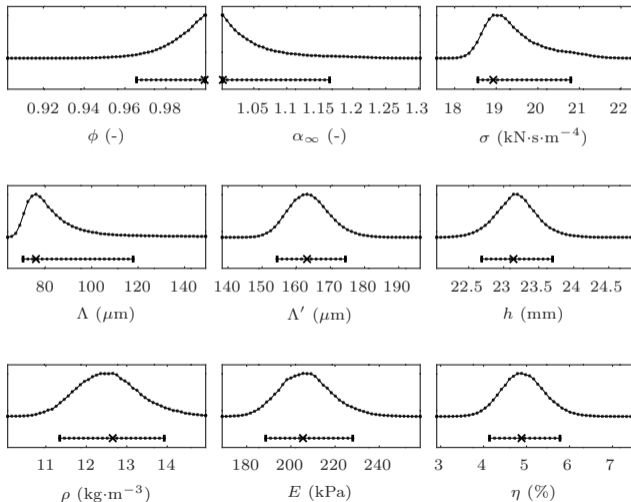
Statistical characterisation of foams

Previous work

Experimental data

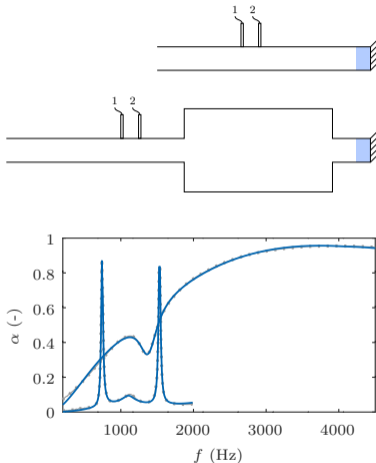


Statistical inversion result

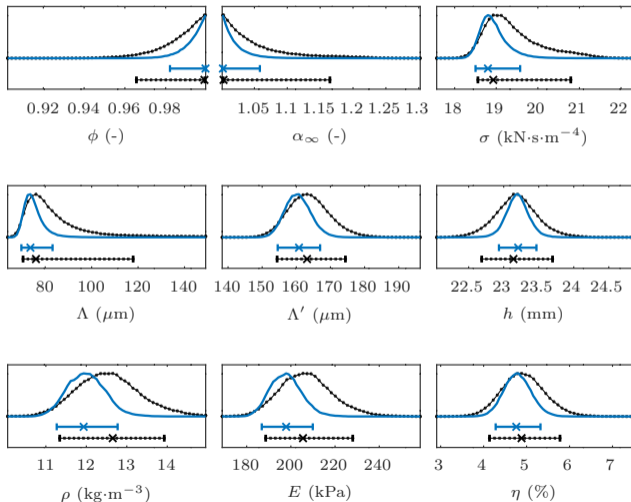


Previous work

Experimental data



Statistical inversion result



Motivation and goal

Motivation

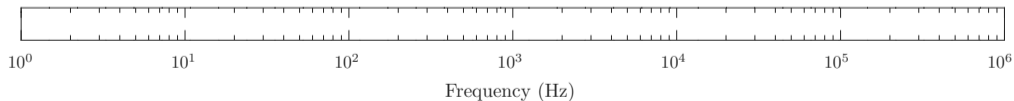
More equations for the same unknowns



Less uncertainty

Idea

Combine measurement data from
different frequency ranges



Motivation and goal

Motivation

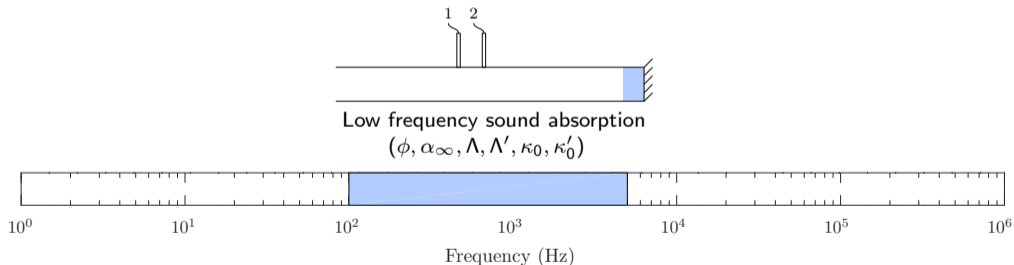
More equations for the same unknowns



Less uncertainty

Idea

Combine measurement data from different frequency ranges



Motivation and goal

Motivation

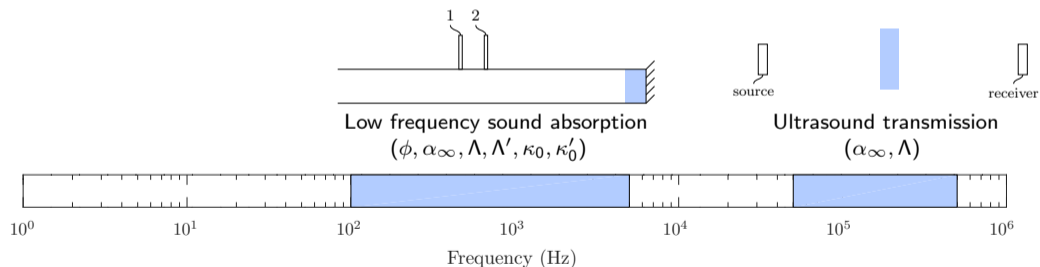
More equations for the same unknowns



Less uncertainty

Idea

Combine measurement data from different frequency ranges



Model and setup

Inverse problem

Numerical tests

Experiments

Closing remarks

Model and setup

Inverse problem

Numerical tests

Experiments

Closing remarks

Model and setup

Porous material model

Johnson-Champoux-Allard-Lafarge model

e.g. with log-normal pore-size distribution parameterisation*

$$\xi = e^{(\sigma_s \log 2)^2}$$

$$\alpha_\infty = \xi^4$$

$$\Lambda = \bar{s} \xi^{-5/2}$$

$$\Lambda' = \bar{s} \xi^{3/2}$$

$$\kappa_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^{-6}$$

$$\kappa'_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^6$$

Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$

* Horoshenkov, Hurrell, Groby. JASA 145 (2019) 2512

Model and setup

Porous material model

Johnson-Champoux-Allard-Lafarge model

e.g. with log-normal pore-size distribution parameterisation*

$$\xi = e^{(\sigma_s \log 2)^2}$$

$$\alpha_\infty = \xi^4$$

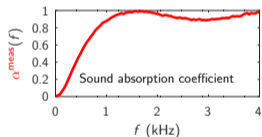
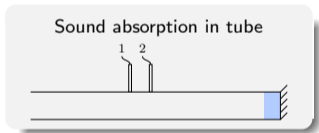
$$\Lambda = \bar{s} \xi^{-5/2}$$

$$\Lambda' = \bar{s} \xi^{3/2}$$

$$\kappa_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^{-6}$$

$$\kappa'_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^6$$

Experimental setups



Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$

Measurement data

$$\alpha^{\text{meas}}(\omega)$$

* Horoshenkov, Hurrell, Groby. JASA 145 (2019) 2512

Model and setup

Porous material model

Johnson-Champoux-Allard-Lafarge model

e.g. with log-normal pore-size distribution parameterisation*

$$\xi = e^{(\sigma_s \log 2)^2}$$

$$\alpha_\infty = \xi^4$$

$$\Lambda = \bar{s} \xi^{-5/2}$$

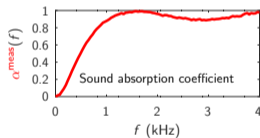
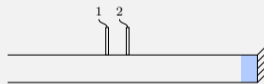
$$\Lambda' = \bar{s} \xi^{3/2}$$

$$\kappa_0 = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^{-6}$$

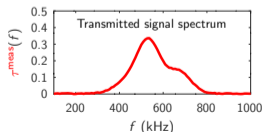
$$\kappa_0' = \frac{\bar{s}^2 \phi}{8\alpha_\infty} \xi^6$$

Experimental setups

Sound absorption in tube



Ultrasound transmission



Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$

Measurement data

$$\alpha^{\text{meas}}(\omega)$$

$$\tau^{\text{meas}}(\omega)$$

* Horoshenkov, Hurrell, Groby. JASA 145 (2019) 2512

Model and setup

Inverse problem

Numerical tests

Experiments

Closing remarks

Statistical model

$$\alpha^{\text{meas}}(\omega) = \alpha(\omega, \mathbf{x}) + \varepsilon(\sigma_\varepsilon)$$



measurement



model



error

\mathbf{x} : unknown parameters
 σ_ε^2 : unknown error variance

Statistical model

$$\alpha^{\text{meas}}(\omega) - \alpha(\omega, \mathbf{x}) = \varepsilon(\sigma_\varepsilon)$$



measurement



model



error

\mathbf{x} : unknown parameters
 σ_ε^2 : unknown error variance

Statistical model

$$\alpha^{\text{meas}}(\omega) - \alpha(\omega, \mathbf{x}) = \varepsilon(\sigma_\varepsilon)$$



measurement



model



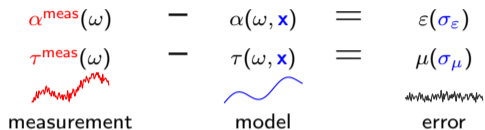
error

\mathbf{x} : unknown parameters
 σ_ε^2 : unknown error variance

Likelihood:

$$P(\alpha^{\text{meas}} | \mathbf{x}) = P_\varepsilon$$

Statistical model



\mathbf{x} : unknown parameters

σ_ε^2 : unknown error variance

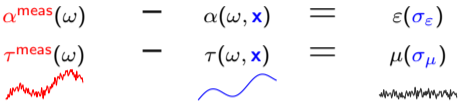
σ_μ^2 : unknown error variance

Likelihood:

$$P(\alpha^{\text{meas}} | \mathbf{x}) = P_\varepsilon$$

$$P(\tau^{\text{meas}} | \mathbf{x}) = P_\mu$$

Statistical model

$$\begin{array}{rcl}
 \alpha^{\text{meas}}(\omega) & - & \alpha(\omega, \mathbf{x}) = \varepsilon(\sigma_\varepsilon) \\
 \tau^{\text{meas}}(\omega) & - & \tau(\omega, \mathbf{x}) = \mu(\sigma_\mu) \\
 \text{measurement} & & \text{model} \quad \text{error}
 \end{array}$$


\mathbf{x} : unknown parameters

σ_ε^2 : unknown error variance

σ_μ^2 : unknown error variance

Likelihood:

$$P(\alpha^{\text{meas}} | \mathbf{x}) = P_\varepsilon$$

$$P(\tau^{\text{meas}} | \mathbf{x}) = P_\mu$$

Posterior:

$$\Rightarrow P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) \propto \underbrace{P(\alpha^{\text{meas}} | \mathbf{x})}_{\text{Likelihood}} \underbrace{P(\tau^{\text{meas}} | \mathbf{x})}_{\text{Likelihood}} \underbrace{P(\mathbf{x})}_{\text{Prior}}$$

Statistical model

$$\begin{array}{rcl}
 \alpha^{\text{meas}}(\omega) & - & \alpha(\omega, \mathbf{x}) = \varepsilon(\sigma_\varepsilon) \\
 \tau^{\text{meas}}(\omega) & - & \tau(\omega, \mathbf{x}) = \mu(\sigma_\mu) \\
 \text{measurement} & & \text{model} \quad \text{error} \\
 & & \text{(Gaussian)}
 \end{array}$$

\mathbf{x} : unknown parameters

σ_ε^2 : unknown error variance

σ_μ^2 : unknown error variance

Likelihood:

$$P(\alpha^{\text{meas}} | \mathbf{x}) = P_\varepsilon = \frac{\exp\left(-\frac{1}{2} \varepsilon^T \Gamma_\varepsilon^{-1} \varepsilon\right)}{\sqrt{(2\pi)^M \det(\Gamma_\varepsilon)}}$$

Posterior:

$$P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) \propto \underbrace{P(\alpha^{\text{meas}} | \mathbf{x})}_{\text{Likelihood}} \underbrace{P(\tau^{\text{meas}} | \mathbf{x})}_{\text{Likelihood}} \underbrace{P(\mathbf{x})}_{\text{Prior}}$$

$$P(\tau^{\text{meas}} | \mathbf{x}) = P_\mu = \frac{\exp\left(-\frac{1}{2} \mu^T \Gamma_\mu^{-1} \mu\right)}{\sqrt{(2\pi)^N \det(\Gamma_\mu)}}$$

Statistical model

$$\begin{array}{rcl}
 \alpha^{\text{meas}}(\omega) & - & \alpha(\omega, \mathbf{x}) = \varepsilon(\sigma_\varepsilon) \\
 \tau^{\text{meas}}(\omega) & - & \tau(\omega, \mathbf{x}) = \mu(\sigma_\mu) \\
 \text{measurement} & & \text{model} \\
 \text{measurement} & & \text{error} \\
 & & \text{(Gaussian)}
 \end{array}$$

\mathbf{x} : unknown parameters

σ_ε^2 : unknown error variance

σ_μ^2 : unknown error variance

$P(\mathbf{x})$: uniform prior

Likelihood:

$$P(\alpha^{\text{meas}}|\mathbf{x}) = P_\varepsilon = \frac{\exp\left(-\frac{1}{2}\varepsilon^T \Gamma_\varepsilon^{-1} \varepsilon\right)}{\sqrt{(2\pi)^M \det(\Gamma_\varepsilon)}}$$

Posterior:

$$P(\mathbf{x}|\alpha^{\text{meas}}, \tau^{\text{meas}}) \propto \underbrace{P(\alpha^{\text{meas}}|\mathbf{x})}_{\text{Likelihood}} \underbrace{P(\tau^{\text{meas}}|\mathbf{x})}_{\text{Likelihood}} \underbrace{P(\mathbf{x})}_{\text{Prior}}$$

$$P(\tau^{\text{meas}}|\mathbf{x}) = P_\mu = \frac{\exp\left(-\frac{1}{2}\mu^T \Gamma_\mu^{-1} \mu\right)}{\sqrt{(2\pi)^N \det(\Gamma_\mu)}}$$

$$\propto \frac{\exp\left(-\frac{\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2}{2\sigma_\varepsilon^2} - \frac{\sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2}{2\sigma_\mu^2}\right)}{\sqrt{(2\pi\sigma_\varepsilon^2\sigma_\mu^2)^{(M+N)}}} P(\mathbf{x})$$

Solution strategy

Posterior

$$P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) = \frac{\exp\left(-\frac{\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2}{2\sigma_\epsilon^2} - \frac{\sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2}{2\sigma_\mu^2}\right)}{\sqrt{(2\pi\sigma_\epsilon^2\sigma_\mu^2)^{(M+N)}}} P(\mathbf{x})$$

Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s, \sigma_\epsilon, \sigma_\mu\}$$

(Hyperparameters σ_ϵ and σ_μ have uniform hyperpriors)

Solution strategy

Posterior

$$P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) = \frac{\exp\left(-\frac{\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2}{2\sigma_\epsilon^2} - \frac{\sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2}{2\sigma_\mu^2}\right)}{\sqrt{(2\pi\sigma_\epsilon^2\sigma_\mu^2)^{(M+N)}}} P(\mathbf{x})$$

Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s, \sigma_\epsilon, \sigma_\mu\}$$

(Hyperparameters σ_ϵ and σ_μ have uniform hyperpriors)

1. Sampling of the posterior

Metropolis-Hastings algorithm with adaptive proposal*

⇒ Markov chain = approximation of target distribution

* Haario, Saksman, Tamminen. Bernoulli 7 (2001) 223-242

Solution strategy

Posterior

$$P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) = \frac{\exp\left(-\frac{\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2}{2\sigma_\epsilon^2} - \frac{\sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2}{2\sigma_\mu^2}\right)}{\sqrt{(2\pi\sigma_\epsilon^2\sigma_\mu^2)^{(M+N)}}} P(\mathbf{x})$$

Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s, \sigma_\epsilon, \sigma_\mu\}$$

(Hyperparameters σ_ϵ and σ_μ have uniform hyperpriors)

0. Initial estimate

Deterministic solution using differential evolution[†]

$$\mathbf{x}^{(\text{init})} = \arg \min_{\mathbf{x}: P(\mathbf{x})} \left(\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2, \sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2 \right)$$

1. Sampling of the posterior

Metropolis-Hastings algorithm with adaptive proposal*

⇒ Markov chain = approximation of target distribution

[†] Storn and Price. J Global Optim 11 (1997) 341-359

* Haario, Saksman, Tamminen. Bernoulli 7 (2001) 223-242

Solution strategy

Posterior

$$P(\mathbf{x} | \alpha^{\text{meas}}, \tau^{\text{meas}}) = \frac{\exp\left(-\frac{\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2}{2\sigma_\epsilon^2} - \frac{\sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2}{2\sigma_\mu^2}\right)}{\sqrt{(2\pi\sigma_\epsilon^2\sigma_\mu^2)^{(M+N)}}} P(\mathbf{x})$$

0. Initial estimate

Deterministic solution using differential evolution[†]

$$\mathbf{x}^{(\text{init})} = \arg \min_{\mathbf{x}: P(\mathbf{x})} \left(\sum_{m=1}^M |\alpha_m^{\text{meas}} - \alpha_m(\mathbf{x})|^2, \sum_{n=1}^N |\tau_n^{\text{meas}} - \tau_n(\mathbf{x})|^2 \right)$$

1. Sampling of the posterior

Metropolis-Hastings algorithm with adaptive proposal*

⇒ Markov chain = approximation of target distribution

Unknowns

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s, \sigma_\epsilon, \sigma_\mu\}$$

(Hyperparameters σ_ϵ and σ_μ have uniform hyperpriors)

2. Post-processing

Point estimates, e.g.:

- Maximum a posteriori estimate $\mathbf{x}^{(\text{MAP})}$
- Conditional mean estimate $\mathbf{x}^{(\text{CM})}$
- Median estimate $\mathbf{x}^{(\text{med})}$

Uncertainty ranges:

- Credible intervals (e.g. 95%)

[†] Storn and Price. J Global Optim 11 (1997) 341-359

* Haario, Saksman, Tamminen. Bernoulli 7 (2001) 223-242

Model and setup

Inverse problem

Numerical tests

Experiments

Closing remarks

Numerical example 1: compatible model

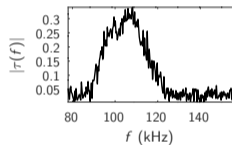
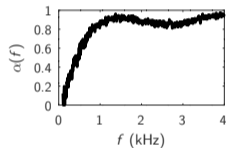
Simulated material (thickness = 40 mm)
Johnson-Champoux-Allard-Lafarge equivalent fluid

ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	2	100 μm	2	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$

Unknowns

Pore-size distribution model parameters

$$\mathbf{x} = \{\phi, \bar{\sigma}, \sigma_s\}$$



Numerical example 1: compatible model

Simulated material (thickness = 40 mm)
Johnson-Champoux-Allard-Lafarge equivalent fluid

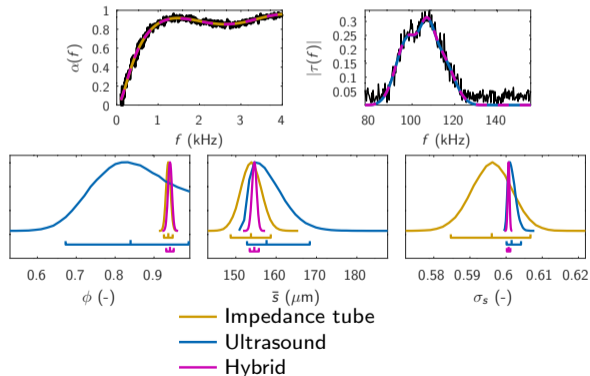
ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	2	100 μm	2	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$

Unknowns

Pore-size distribution model parameters

$$\mathbf{x} = \{\phi, \bar{\sigma}, \sigma_s\}$$

Results



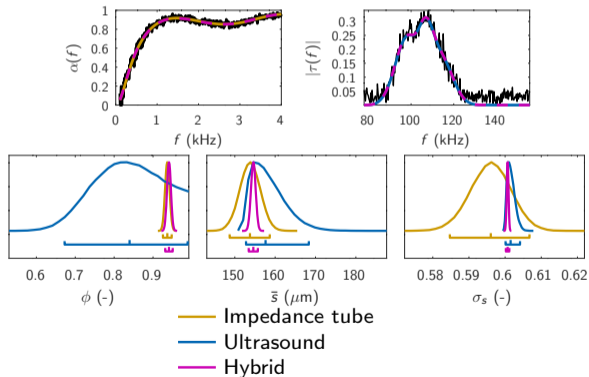
Numerical example 1: compatible model

Simulated material (thickness = 40 mm)

Johnson-Champoux-Allard-Lafarge equivalent fluid

ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	2	100 μm	2	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$

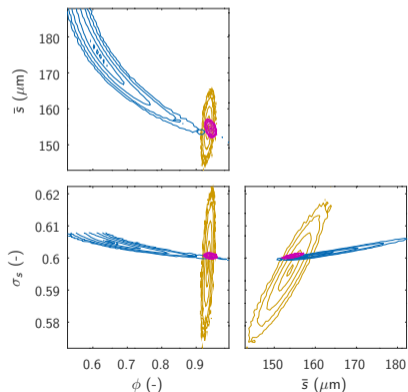
Results



Unknowns

Pore-size distribution model parameters

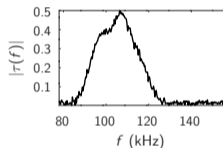
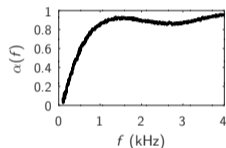
$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$



Numerical example 2: incompatible model

Simulated material (thickness = 40 mm)
Johnson-Champoux-Allard-Lafarge equivalent fluid

ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	1.9	100 μm	2.1	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$



Unknowns

Pore-size distribution model parameters

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$

Numerical example 2: incompatible model

Simulated material (thickness = 40 mm)
Johnson-Champoux-Allard-Lafarge equivalent fluid

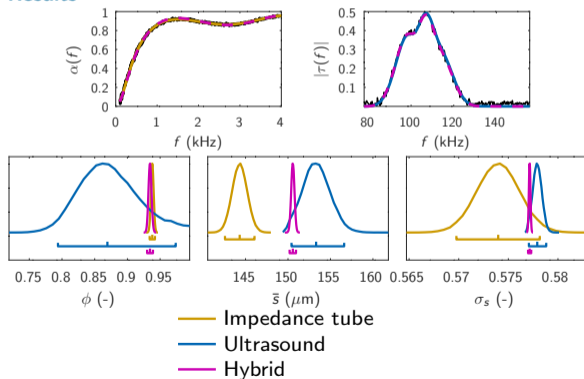
ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	1.9	100 μm	2.1	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$

Unknowns

Pore-size distribution model parameters

$$\mathbf{x} = \{\phi, \bar{\sigma}, \sigma_s\}$$

Results

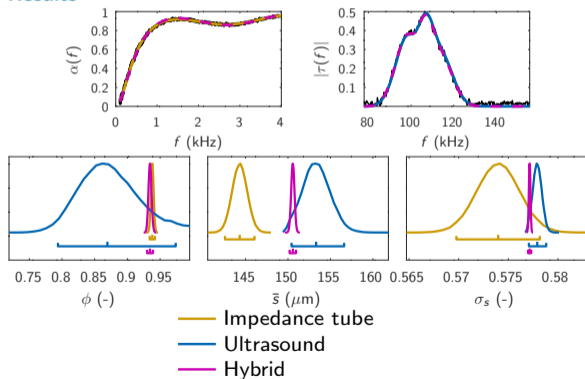


Numerical example 2: incompatible model

Simulated material (thickness = 40 mm)
Johnson-Champoux-Allard-Lafarge equivalent fluid

ϕ	α_∞	Λ	Λ'/Λ	κ_0	κ'_0
0.95	1.9	100 μm	2.1	$0.5 \cdot 10^{-9} \text{m}^2$	$4 \cdot 10^{-9} \text{m}^2$

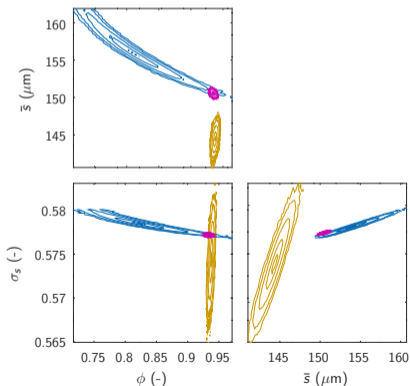
Results



Unknowns

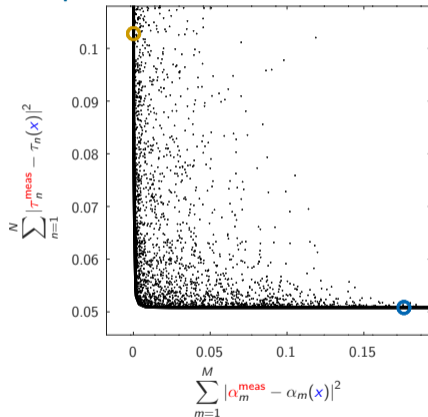
Pore-size distribution model parameters

$$\mathbf{x} = \{\phi, \bar{s}, \sigma_s\}$$



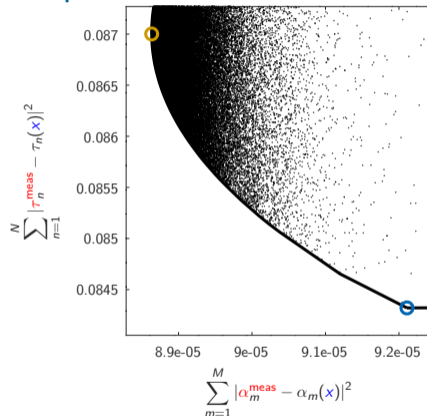
Pareto front

Example 1



Model is **compatible** with data

Example 2



Model is **incompatible** with both datasets simultaneously

Model and setup

Inverse problem

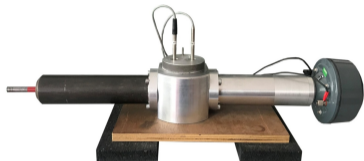
Numerical tests

Experiments

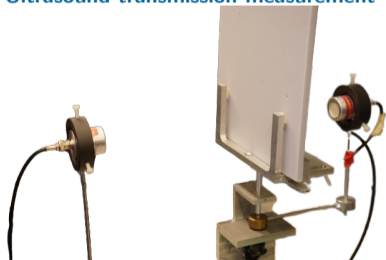
Closing remarks

Setups

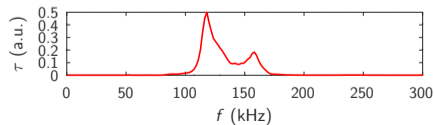
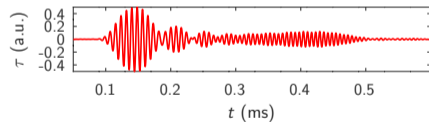
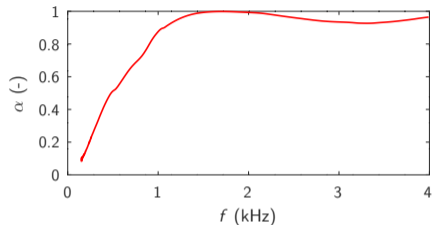
Sound absorption measurement



Ultrasound transmission measurement



Measured data for a sample of melamine foam



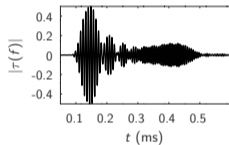
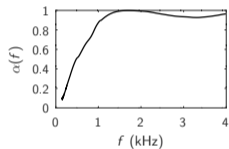
Preliminary experimental results

Tested material

Melamine foam

Thickness = 50 mm (impedance tube)

Thickness = 50 mm (ultrasound)



Unknowns

Johnson-Champoux-Allard-Lafarge model

$$\mathbf{x} = \{\phi, \alpha_{\infty}, \Lambda, \Lambda'/\Lambda, \sigma, \kappa'_0\}$$

Preliminary experimental results

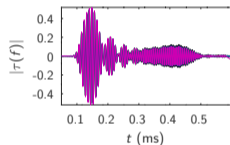
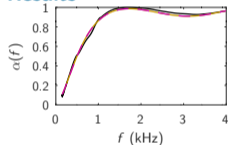
Tested material

Melamine foam

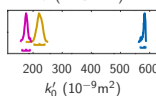
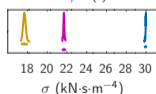
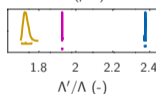
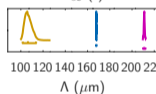
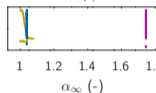
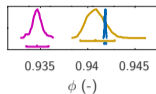
Thickness = 50 mm (impedance tube)

Thickness = 50 mm (ultrasound)

Results



— Impedance tube
— Ultrasound
— Hybrid



Unknowns

Johnson-Champoux-Allard-Lafarge model

$$\mathbf{x} = \{\phi, \alpha_\infty, \Lambda, \Lambda'/\Lambda, \sigma, \kappa'_0\}$$

Preliminary experimental results

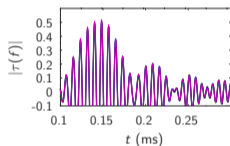
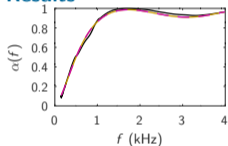
Tested material

Melamine foam

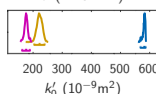
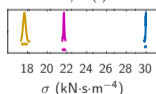
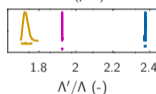
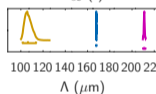
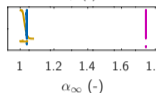
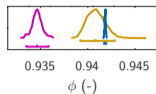
Thickness = 50 mm (impedance tube)

Thickness = 50 mm (ultrasound)

Results



— Impedance tube
— Ultrasound
— Hybrid



Unknowns

Johnson-Champoux-Allard-Lafarge model

$$\mathbf{x} = \{\phi, \alpha_{\infty}, \Lambda, \Lambda' / \Lambda, \sigma, \kappa_0'\}$$

Preliminary experimental results

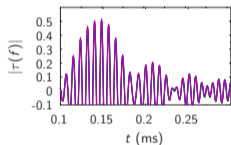
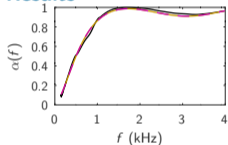
Tested material

Melamine foam

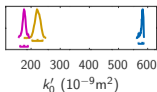
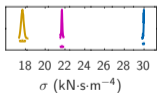
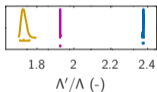
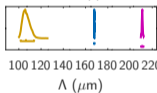
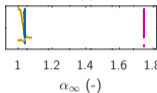
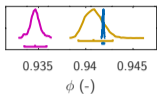
Thickness = 50 mm (impedance tube)

Thickness = 50 mm (ultrasound)

Results



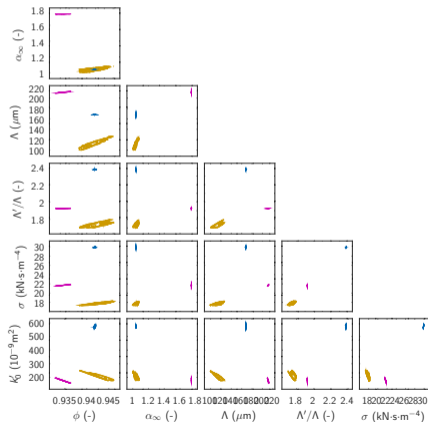
— Impedance tube
— Ultrasound
— Hybrid



Unknowns

Johnson-Champoux-Allard-Lafarge model

$$\mathbf{x} = \{\phi, \alpha_\infty, \Lambda, \Lambda'/\Lambda, \sigma, \kappa'_0\}$$



Preliminary experimental results

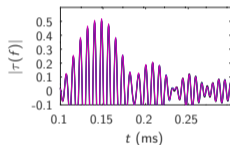
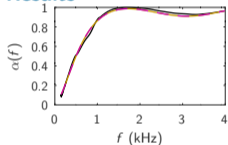
Tested material

Melamine foam

Thickness = 50 mm (impedance tube)

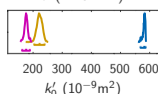
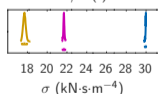
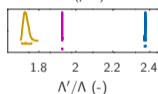
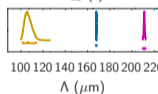
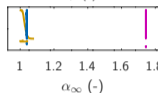
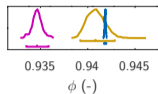
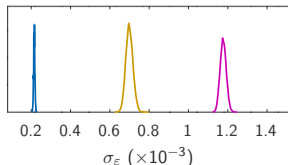
Thickness = 50 mm (ultrasound)

Results



— Impedance tube
— Ultrasound
— Hybrid

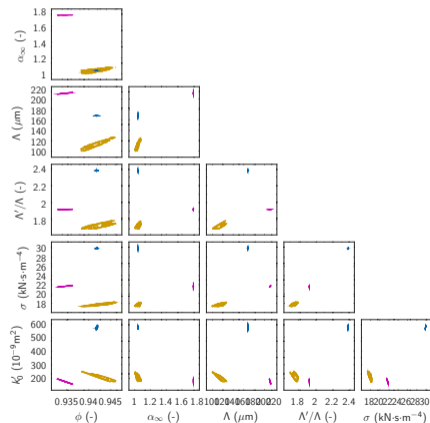
Reconstruction error



Unknowns

Johnson-Champoux-Allard-Lafarge model

$$\mathbf{x} = \{\phi, \alpha_\infty, \Lambda, \Lambda'/\Lambda, \sigma, \kappa_0'\}$$



Model and setup

Inverse problem

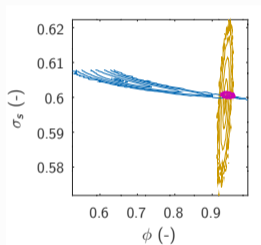
Numerical tests

Experiments

Closing remarks

Closing remarks

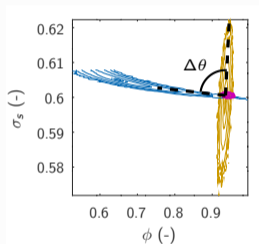
Different datasets allow to look at the inverse problem from different angles



- Impedance tube
- Ultrasound
- Hybrid

Closing remarks

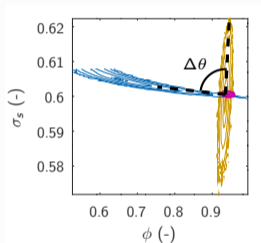
Different datasets allow to look at the inverse problem from different angles
... literally



- Impedance tube
- Ultrasound
- Hybrid

Closing remarks

Different datasets allow to look at the inverse problem from different angles
... literally



- Impedance tube
- Ultrasound
- Hybrid

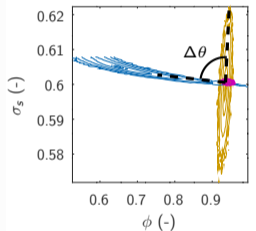
More experimental data

reduces parameter
correlation

may introduce
non-uniqueness

Closing remarks

Different datasets allow to look at the inverse problem from different angles
... literally



— Impedance tube
— Ultrasound
— Hybrid

More experimental data

reduces parameter
correlation

may introduce
non-uniqueness

Future work

- Characterisation of individual samples
⇒ circumvent inhomogeneity
- Multi-objective Bayesian approach for the **design** of locally-resonant media:
Objective vs. **subjective** criteria

Acknowledgements

Dr. Timo Lähivaara
University of Eastern Finland, Kuopio, Finland

Prof. Peter Göransson
KTH Royal Institute of Technology, Stockholm, Sweden



European Commission's Horizon Europe research and innovation programme
Grant agreement No. 101072415