Vibro-Acoustic modelling, simulation and optimisation of anisotropic cellular materials

ROYAL INSTITUTE OF TECHNOLOGY

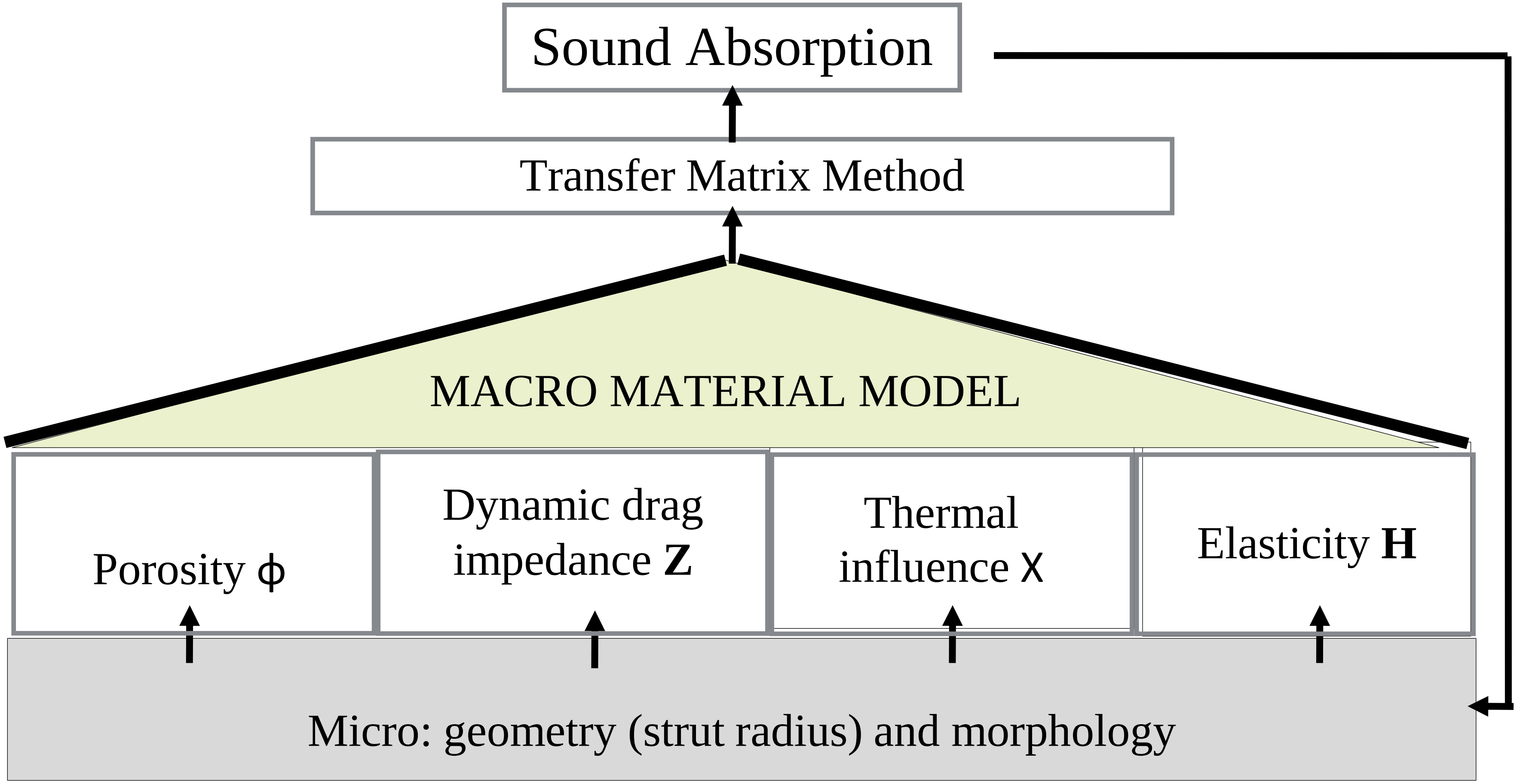
Eva Lundberg, Huina Mao, Mathieu Gaborit, Romain Rumpler, Brad Semeniuk, Peter Göransson

KTH Royal Institute of Technology, Sweden
Volvo CE, Sweden
Laboratoire d'Acoustique de l'Université du Mans (LAUM), France

with gratitude for all the collaborations over the years from
Olivier Dazel, Jacques Cuenca, Juan Pablo Parra Martinez, …

Sorrento SAPEM'23 常熟

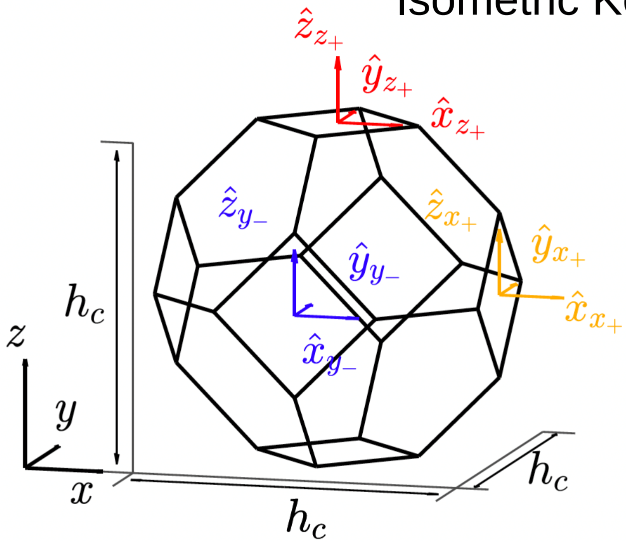
From microstructure to optimisation of macro performance



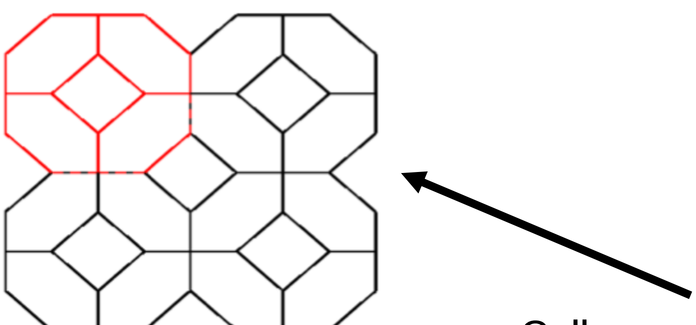
Geometry and morphology

ROYAL INSTITUTE
OF TECHNOLOGY

Isometric Kelvin cell



Iso

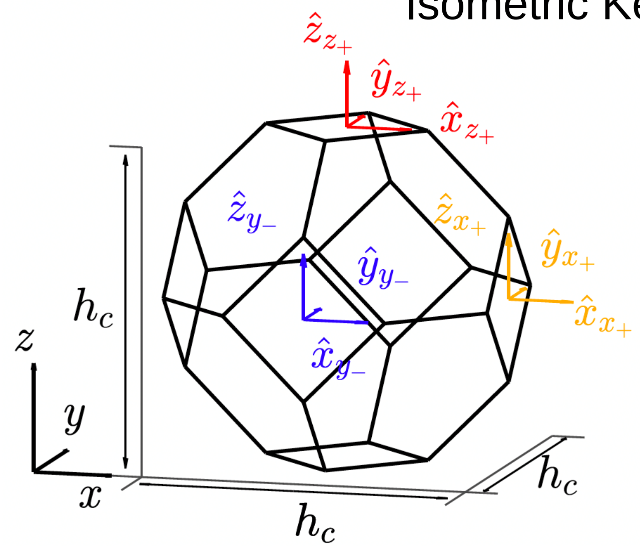


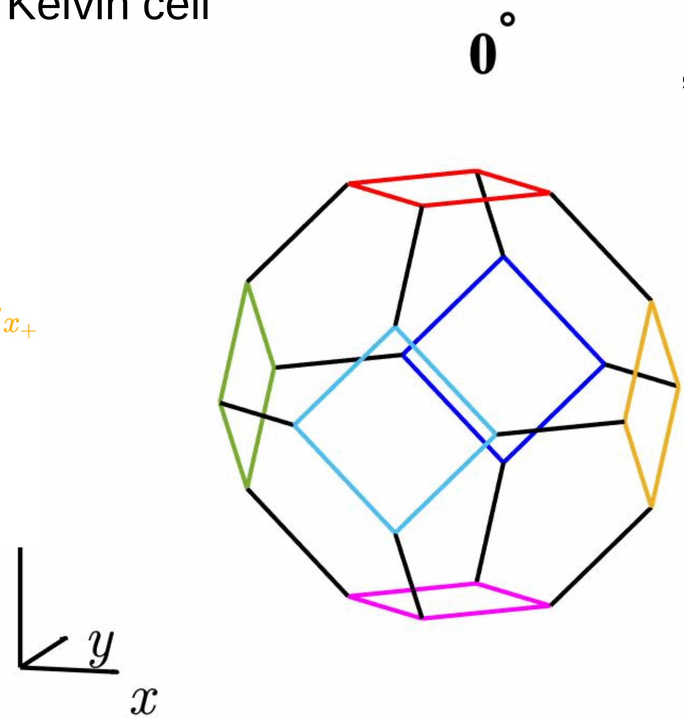
Cell arrays 2 x 2 x 2 - top view

H. Mao et al. Materials & Design, vol. 193, p. 108855, 2020

Geometry and morphology

Isometric Kelvin cell

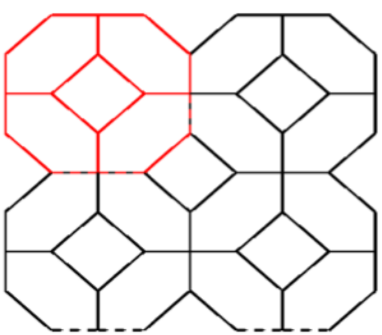




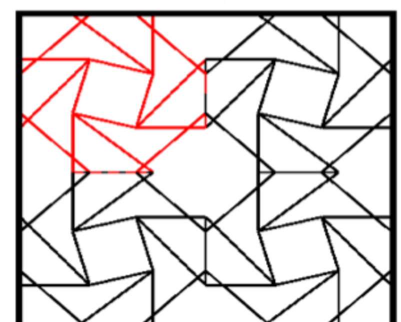
”Do the twist again”

Anti-symmetric

Iso



60°

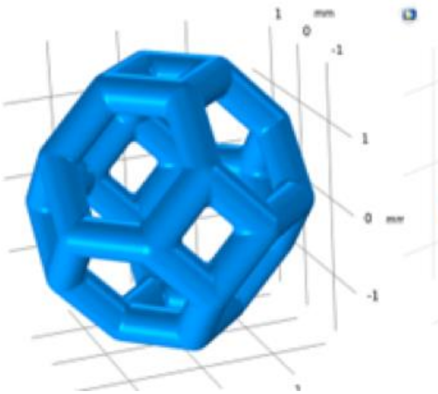


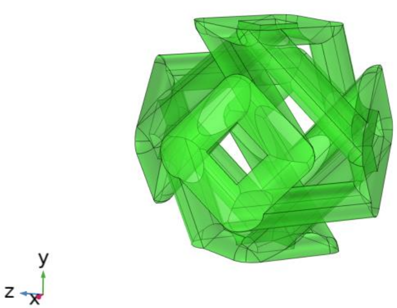
Cell arrays 2 x 2 x 2 - top view

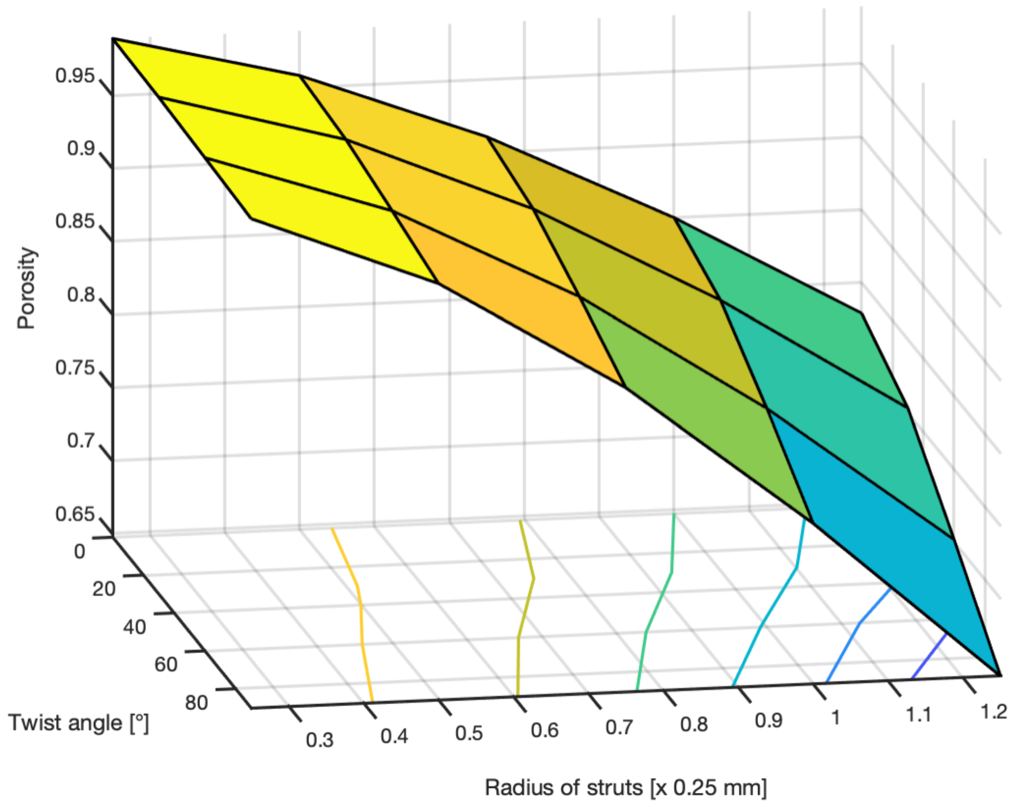
H. Mao et al. Materials & Design, vol. 193, p. 108855, 2020

Porosity

$$\phi = 1 - \frac{Volume_{struts}}{Volume_{cell}}$$







Cell height 3 mm

Thinner struts

Thicker struts

Governing equations

- $\boldsymbol{u}^s$ solid displacement
- $\boldsymbol{u}^f$ fluid displacement
- p pressure
- $\tilde{K}_{eq}$ fluid compressibility
- ρ_s solid constituent density
- ρ_f fluid density
- $\hat{\boldsymbol{\sigma}}^s$ Cauchy stress
- $\boldsymbol{H}$ Hooke's matrix
- $\boldsymbol{\varepsilon}^s$ linear strain

$$\boldsymbol{u}^t = (1-\phi)\boldsymbol{u}^s + \phi \boldsymbol{u}^f$$

$$\nabla \hat{\boldsymbol{\sigma}}^s = -\omega^2 \tilde{\boldsymbol{\rho}}_s \boldsymbol{u}^s - \omega^2 \tilde{\boldsymbol{\tau}} \boldsymbol{u}^t$$

$$-\nabla p = -\omega^2 \tilde{\boldsymbol{\tau}} \boldsymbol{u}^s - \omega^2 \tilde{\boldsymbol{\rho}}_t \boldsymbol{u}^t$$

$$p = -\tilde{K}_{eq} \nabla \cdot \boldsymbol{u}^t$$

$$\hat{\boldsymbol{\sigma}}^s = \boldsymbol{H} \boldsymbol{\epsilon}^s$$

$$\tilde{\boldsymbol{\rho}}_s = (1-\phi)\rho_s \boldsymbol{I} + \frac{(1-\phi)^2}{\phi}\rho_f \boldsymbol{I} - \frac{i\boldsymbol{Z}}{\omega\phi^2}$$

Need to establish models for:

$\boldsymbol{Z}$

$\tilde{K}_{eq}$

$\boldsymbol{H}$

$$\tilde{\boldsymbol{\tau}} = -\frac{(1-\phi)}{\phi}\rho_f \boldsymbol{I} + \frac{i\boldsymbol{Z}}{\omega\phi^2},$$

Cmf $\tilde{\boldsymbol{\rho}}_{eq}\tilde{\gamma}$

$$\tilde{\boldsymbol{\rho}}_t = \frac{\rho_f \boldsymbol{I}}{\phi} - \frac{i\boldsymbol{Z}}{\phi^2\omega}.$$

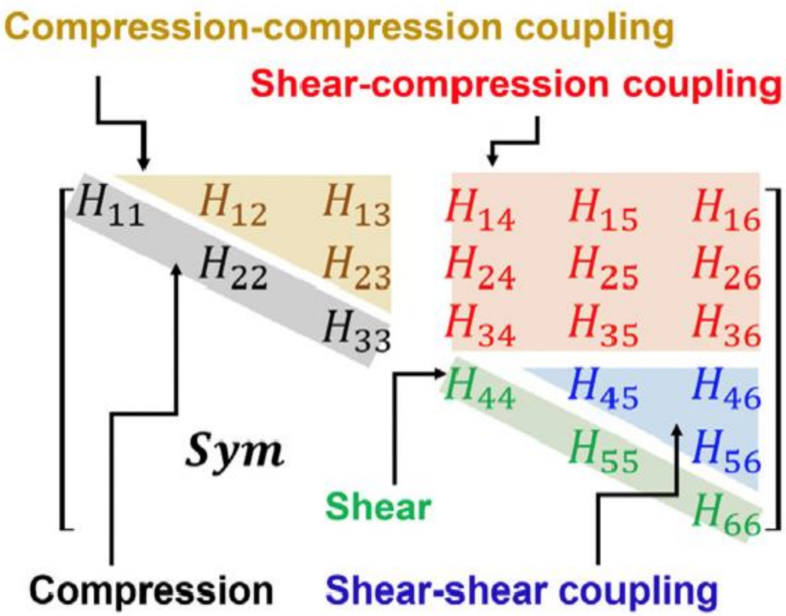
Cmf $\tilde{\boldsymbol{\rho}}_{eq}$

Hooke's tensor linear elasticity - recap

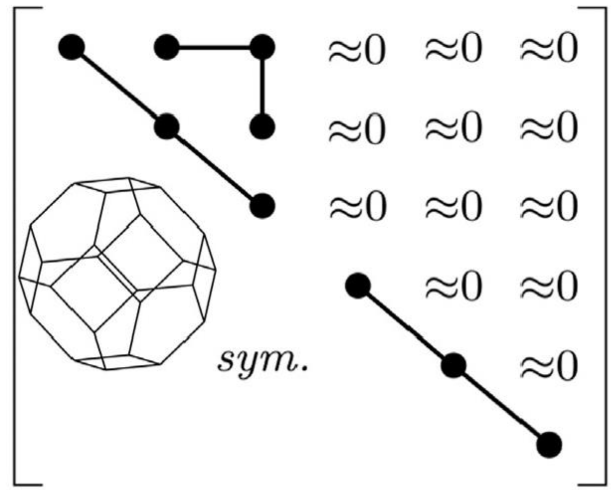
Hooke's law $\boldsymbol{\sigma} = \mathbf{H}\boldsymbol{\varepsilon}$

Cauchy stress $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}\}^T$

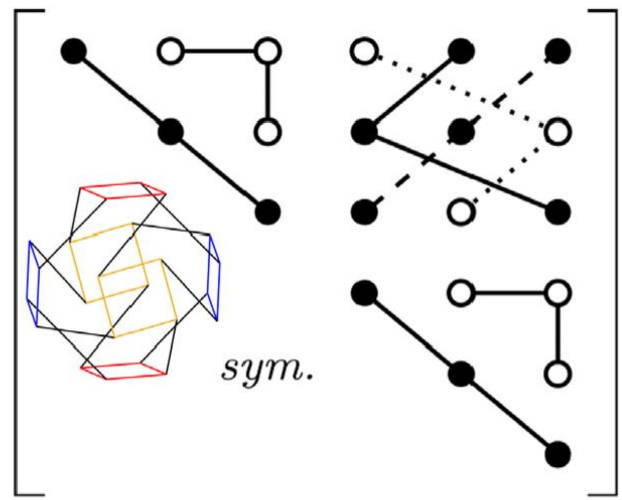
Linear strain $\boldsymbol{\varepsilon} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{12}, 2\varepsilon_{13}, 2\varepsilon_{23}\}^T$



Iso



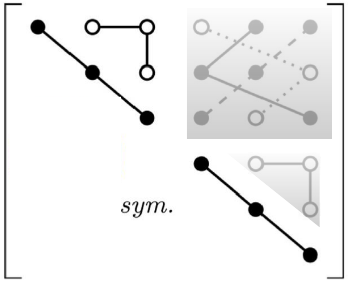
Anti-symmetric twist

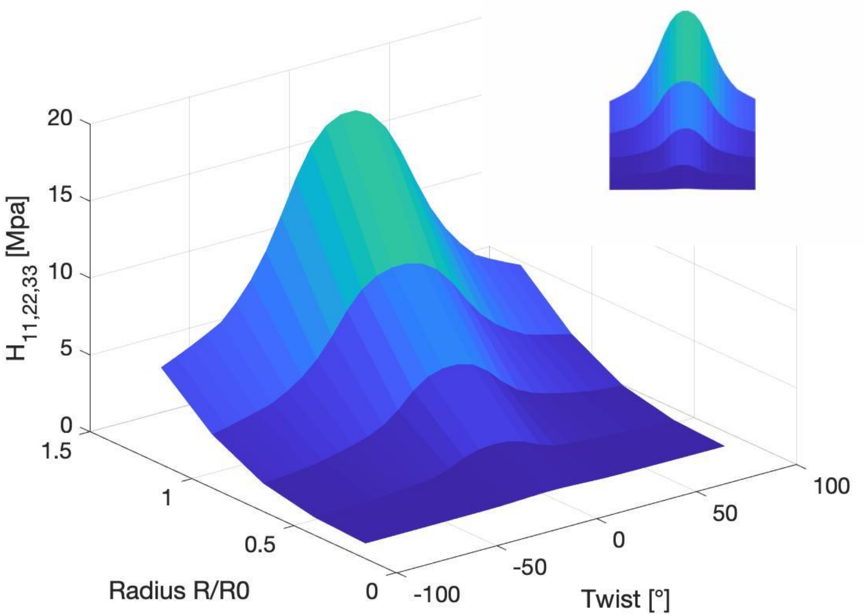


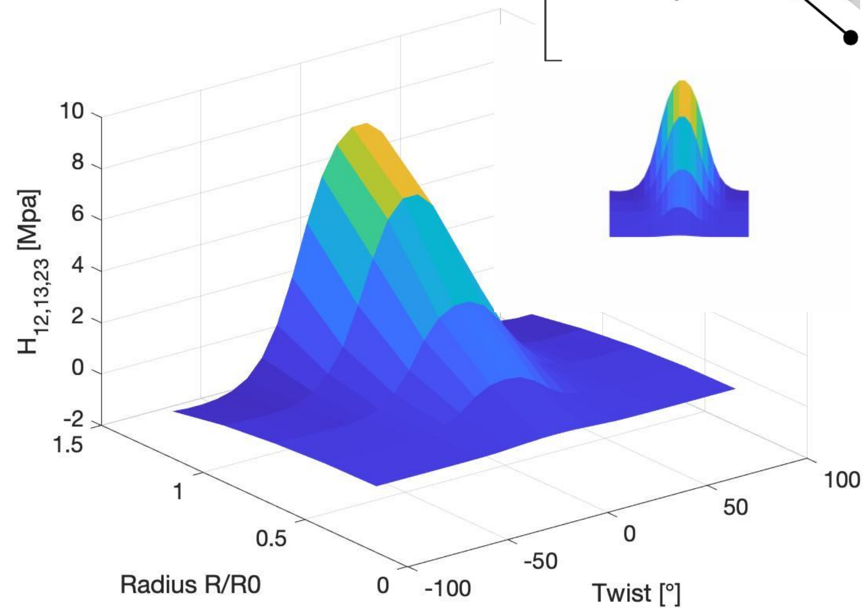
H. Mao et al, Twist, tilt and stretch: From isometric Kelvin cells to anisotropic cellular materials, Mater. Des. 193 (2020) 108855

H. Mao, R. Rumpler, P. Göransson, An inverse method for characterisation of the static elastic Hooke's tensors of solid frame of anisotropic open-cell materials, Int. J. Engrg. Sci. 147 (2020) 103198

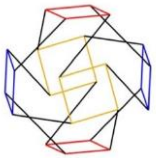
Hooke's tensor variation
compression and shear moduli

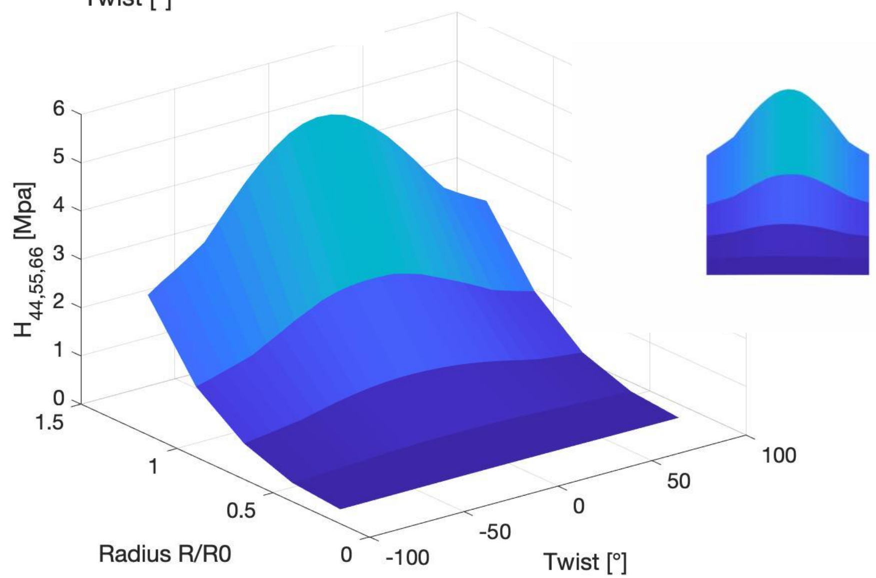






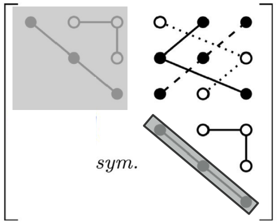
Anti-symmetric twist



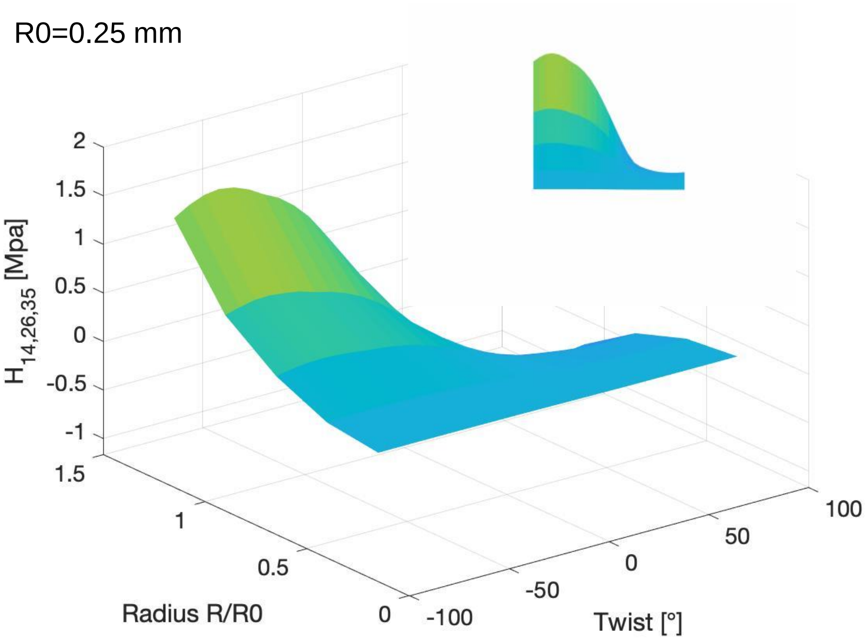


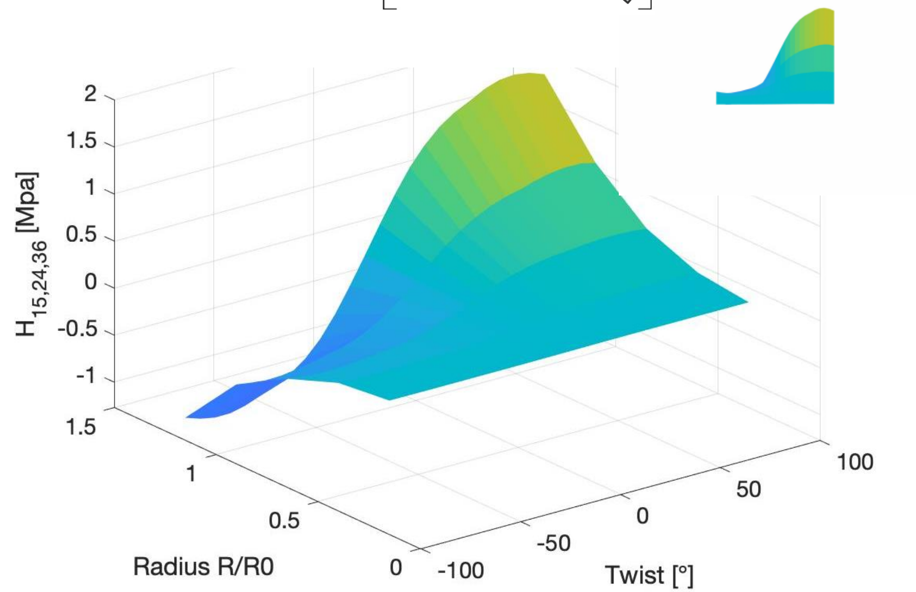
R0=0.25 mm

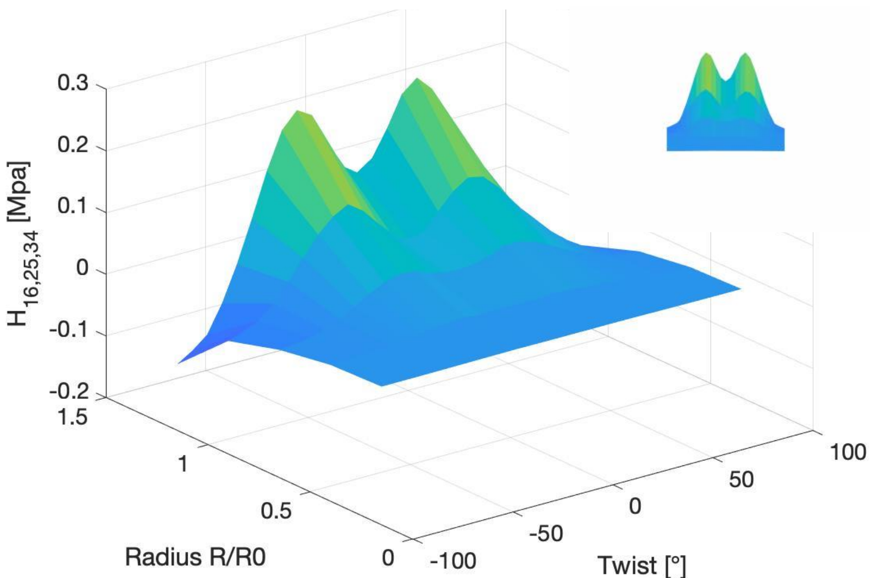
Hooke's tensor variation
compression-shear & shear-shear

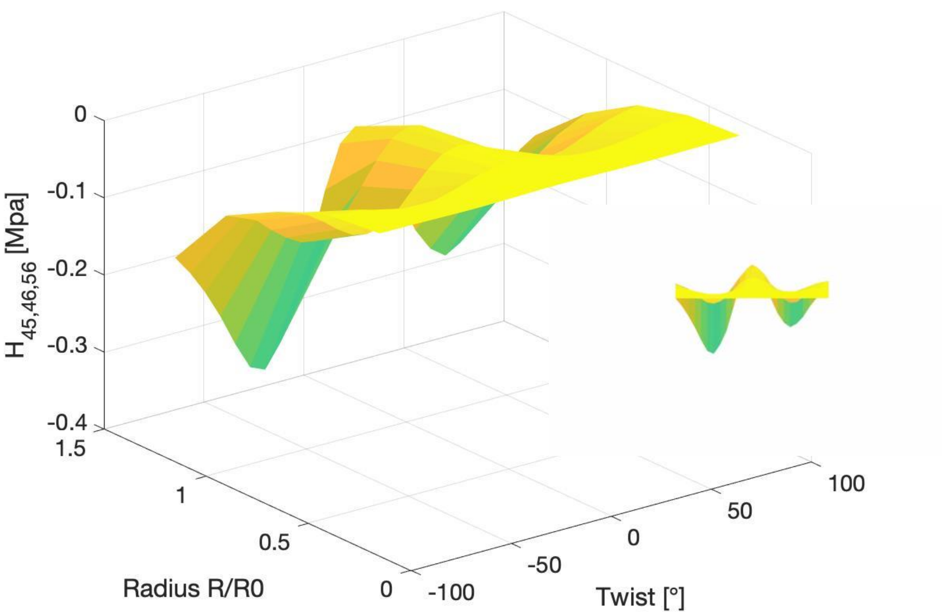


R0=0.25 mm





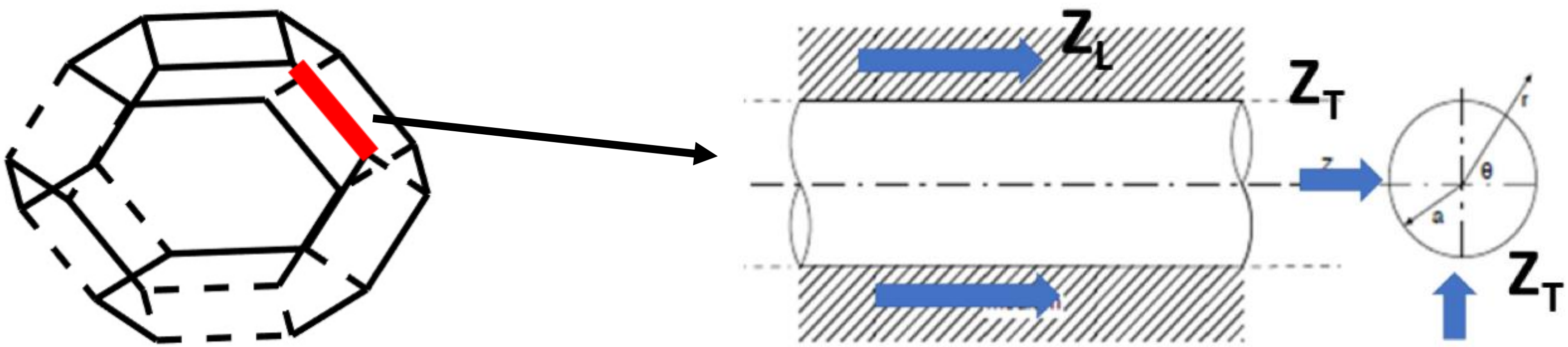




Dynamic drag impedance

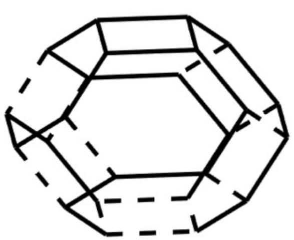
ROYAL INSTITUTE OF TECHNOLOGY

In a local coordinate system

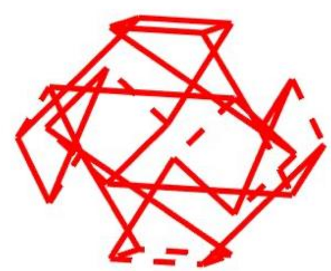


$$\boldsymbol{Z}_{\text{cell}} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} = \frac{1-\phi}{V_s} \sum_{n=1}^{N} \boldsymbol{R_n} \begin{bmatrix} Z_{L_n} & 0 & 0 \\ 0 & Z_{T_n} & 0 \\ 0 & 0 & Z_{T_n} \end{bmatrix} \boldsymbol{R_n^T}.$$

$\boldsymbol{R_n}$
rotation matrix



$$\begin{pmatrix} Z_{iso} & 0 & 0 \\ 0 & Z_{iso} & 0 \\ 0 & 0 & Z_{iso} \end{pmatrix}$$



$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

B. P. Semeniuk, E. Lundberg, and P. Goransson. “Acoustics modelling of open-cell foam materials from microstructure and constitutive properties”. The Journal of the Acoustical Society of America 149.3 (2021), pp. 2016–2026.

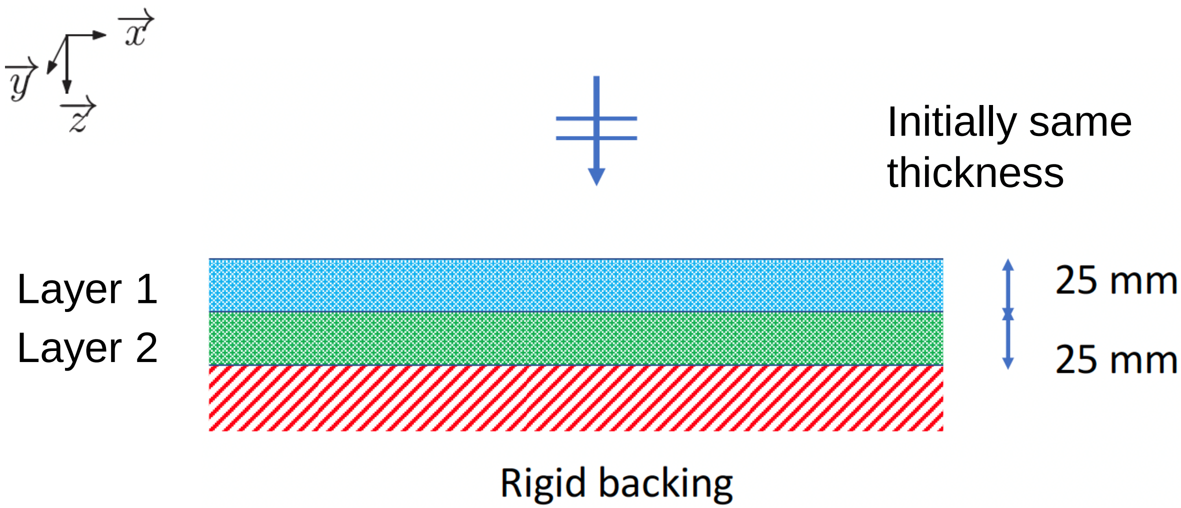
Solving for the absorption

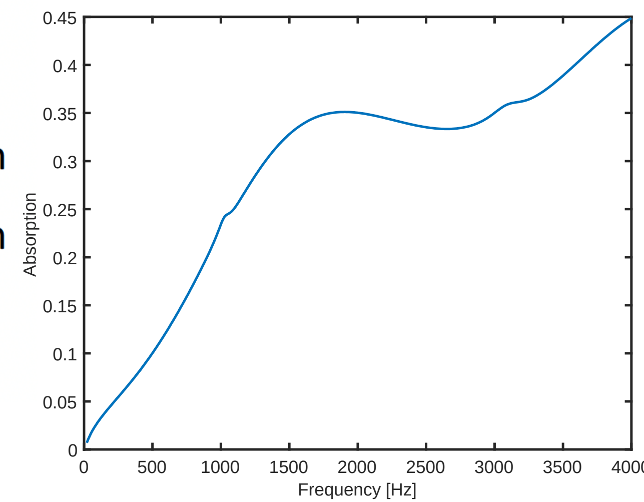
Assume a semi-infinite stack of two layers on a rigid backing, total thickness 50 mm.

Each of the layers is a (possibly) twisted Kelvin cell based equivalent material.

R0=0.25 mm

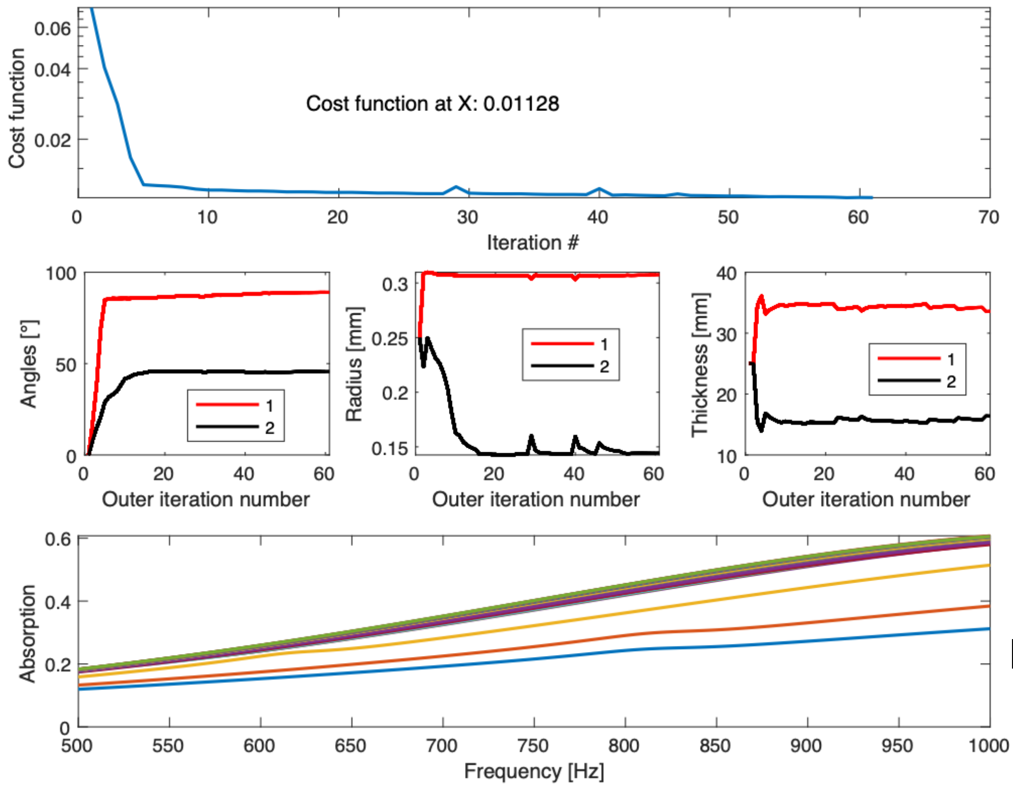
ϕ = 0.85





O Dazel et al. " A stable method to model the acoustic response of multilayered structures". J Applied Physics 113 (2013), p. 83506
J P Parra Martinez et al. "Acoustic analysis of anisotropic poroelastic multilayered systems". J Applied Physics 119.8 (2016), p. 84907.
J P Parra Martinez et al. "Derivation of the state matrix for dynamic analysis of linear homogeneous media". JASA 140.2 (2016), EL218-EL220.

Two layers -
cost function max(Absorption) 500-1000 Hz



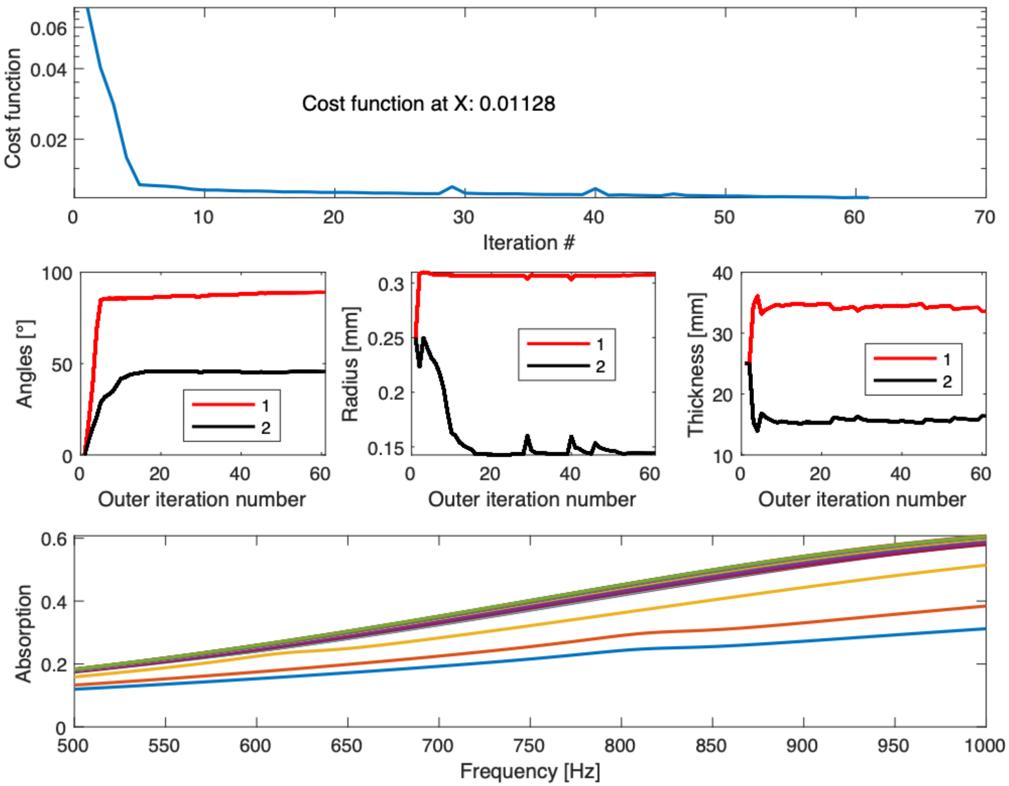
NB 4 design variables, radii and thicknesses

Final solution

Initial solution

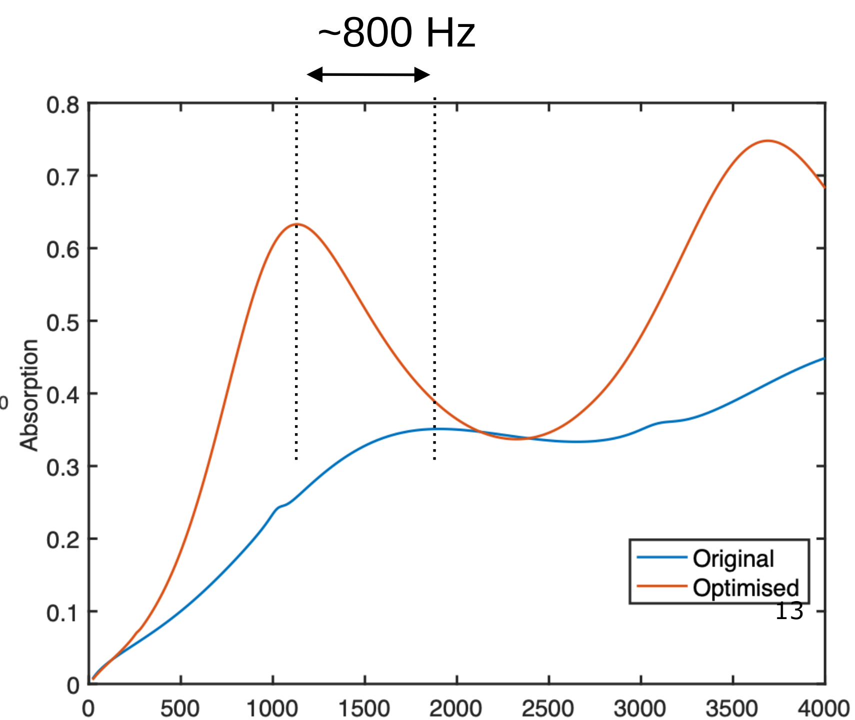
Optimisation step

Two layers -
cost function max(Absorption) 500-1000 Hz



Optimisation step

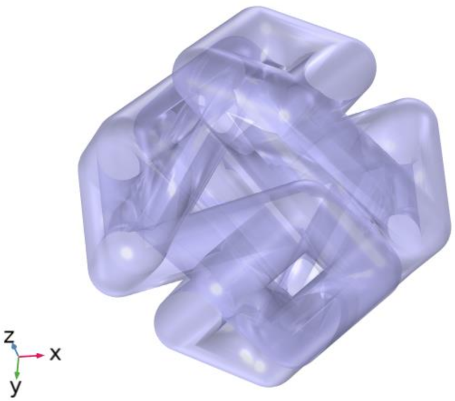
Absorption spectrum

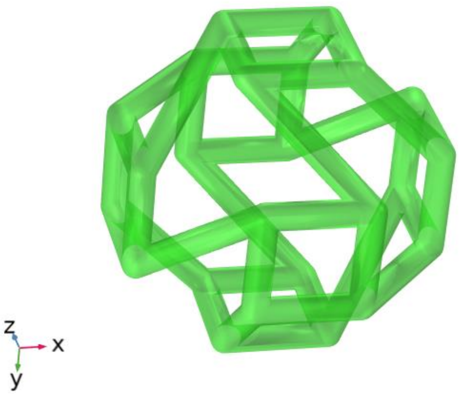


Two layers solution-
max(Absorption) 500-1000 Hz

Layer	Twist °	Radius mm	Thickness mm	Porosity %	Surface density kg/m^2
top	89	0.31	34	66	18.4
bottom	46	0.14	16	93	1.8

Optimal cell microgeometries





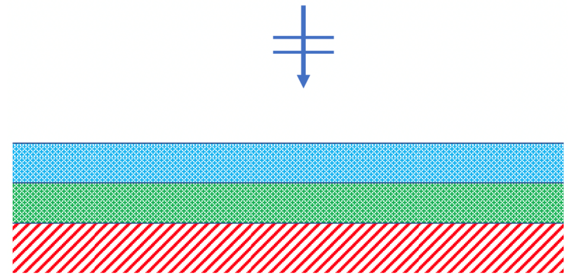
... research where ECOlogy & ECOnomy meet

Optimised anisotropic

Iso

36 mm

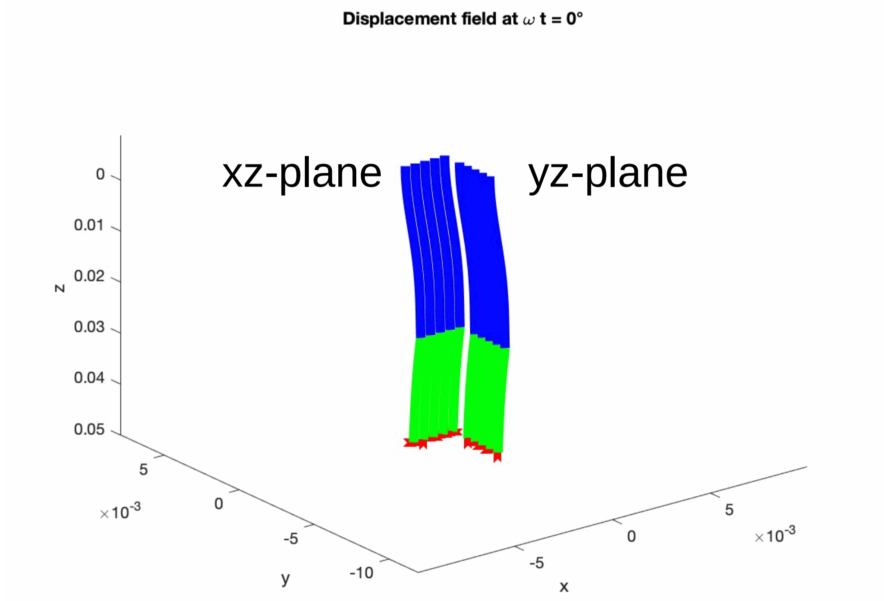
14 mm

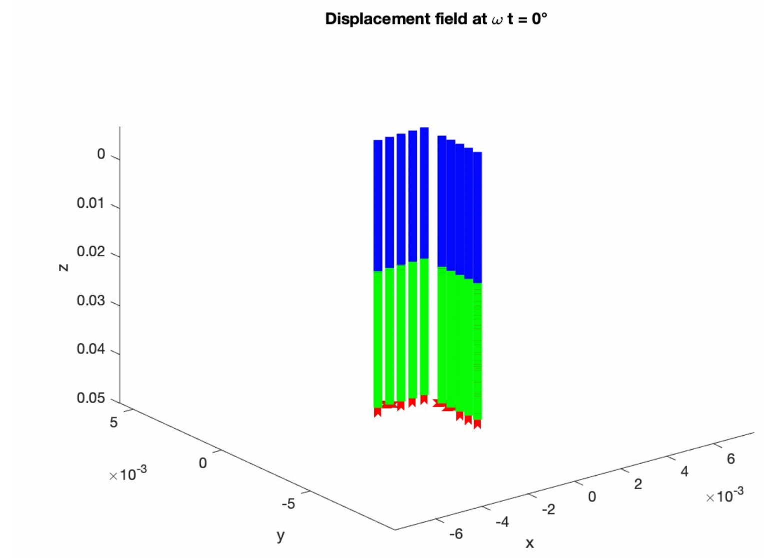


25 mm

25 mm

Rigid backing



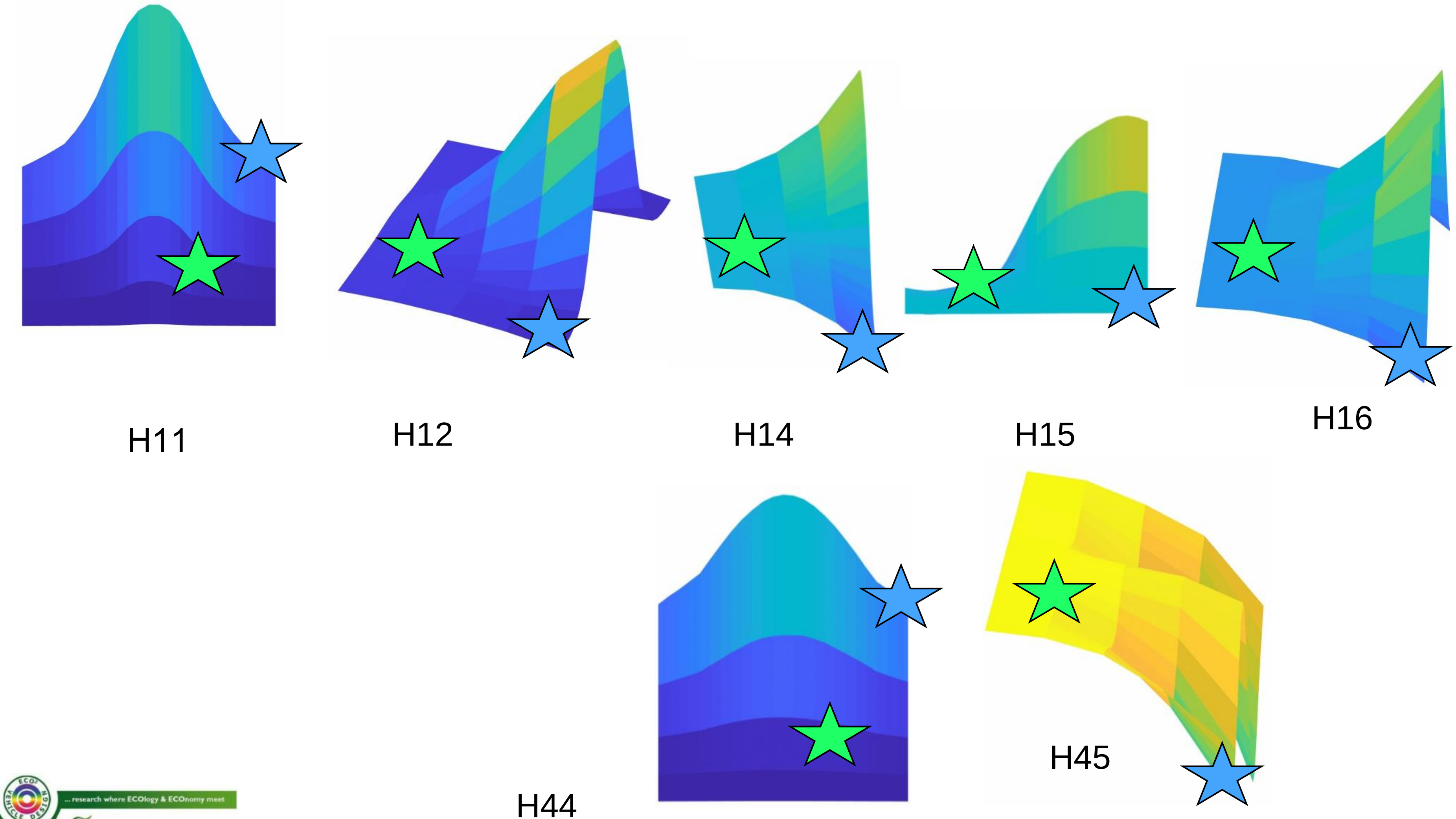


~1100 Hz

~1900 Hz

U = real(u * e^(i ω t)), animation over ωt $\in$ [0 2π]

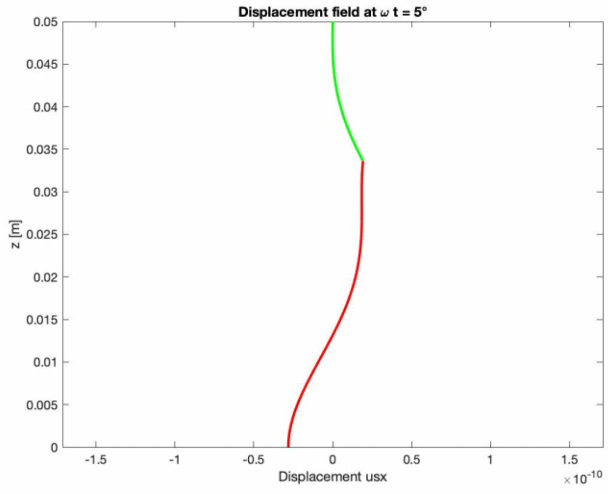
The optimal solutions elastic parameters

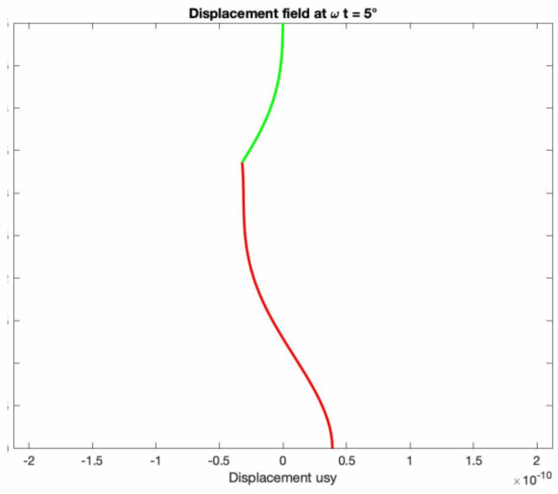


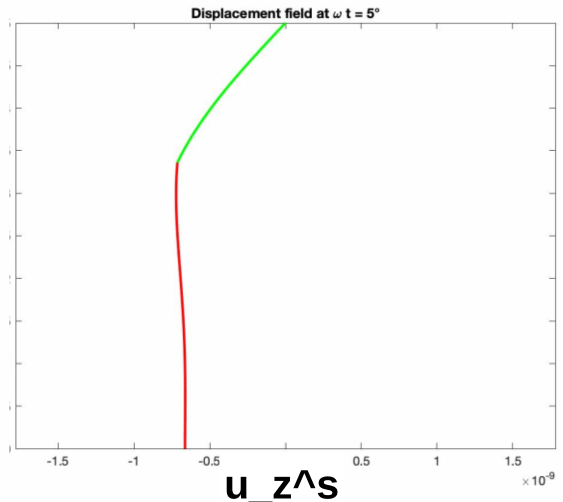
Conclusions

- General framework for acoustic anisotropic poro-elastic material modelling based on an open cell template.
- Microstructure geometry based modelling of elasticity and viscous drag enable meaningful optimisation.
- Optimal vibro-acoustic performance:
  - Resulting cell geometry highly distorted and different between layers
  - A complex interplay between different deformation mechanisms

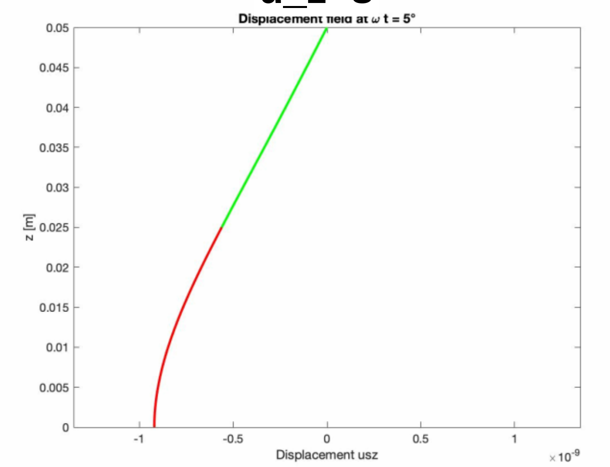
Aniso optimal @ absorption peak 1100 Hz







u_z^s



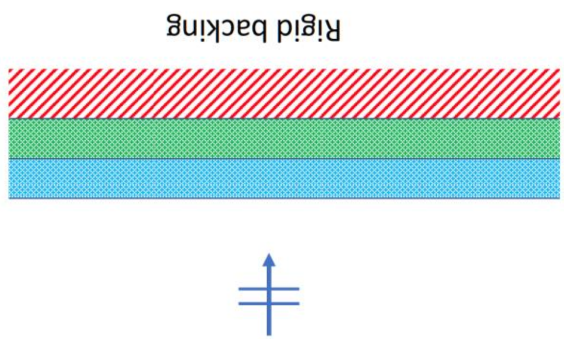
u_x^s

u_y^s

Iso @ absorption peak 1800 Hz

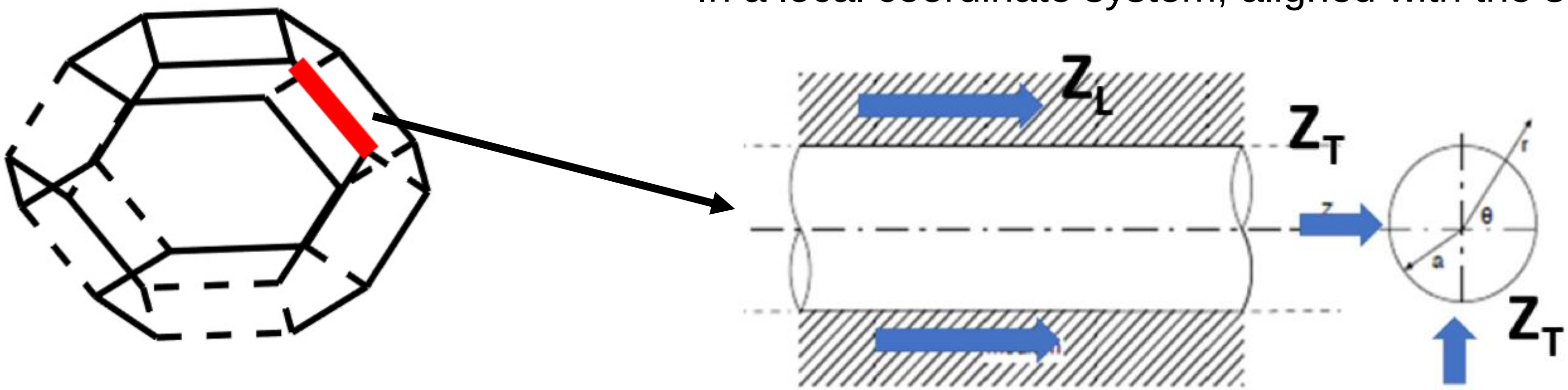
0

0



Dynamic drag impedance

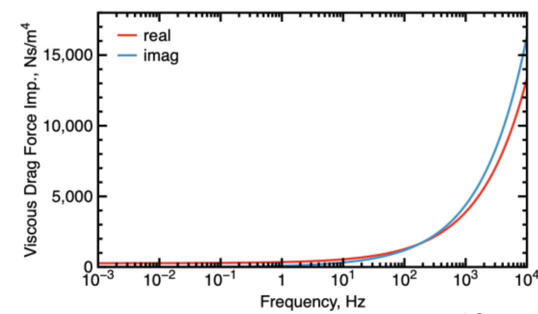
In a local coordinate system, aligned with the strut



$$Z_{Ln} = 2\pi \int_0^{l_n} r_n k_\beta \mu_f \frac{H_1^{(2)}(k_\beta r_n)}{H_0^{(2)}(k_\beta r_n)} dz = 2\pi r_n l_n k_\beta \mu_f \frac{H_1^{(2)}(k_\beta r_n)}{H_0^{(2)}(k_\beta r_n)},$$

$$Z_{Tn} = i\pi r_n^2 l_n \rho_f \omega \left(1 - \frac{4H_1^{(2)}(k_\beta r_n)}{k_\beta r_n H_0^{(2)}(k_\beta r_n)}\right),$$

$$k_\beta = \sqrt{-(i\omega\rho_f/\mu_f}$$



B. P. Semeniuk, E. Lundberg, and P. Goransson. “Acoustics modelling of open-cell foam materials from microstructure and constitutive properties”. The Journal of the Acoustical Society of America 149.3 (2021), pp. 2016–2026.

Hooke's matrix

Normalised with H_{11}

Layer 1 x 5.5 MPa = H_{11}

			ε_{xy}	ε_{xz}	ε_{yz}
1.0000	−0.1320	−0.1320	−0.1995	0.2507	−0.0189
−0.1320	1.0000	−0.1320	0.2507	−0.0189	−0.1995
−0.1320	−0.1320	1.0000	−0.0189	−0.1995	0.2507
−0.1995	0.2507	−0.0189	0.4663	−0.0216	−0.0216
0.2507	−0.0189	−0.1995	−0.0216	0.4663	−0.0216
−0.0189	−0.1995	0.2507	−0.0216	−0.0216	0.4663

Layer 2 x 0.9 MPa = H_{11}

1.0000	0.0126	0.0126	−0.1132	0.1451	0.0175
0.0126	1.0000	0.0126	0.1451	0.0175	−0.1132
0.0126	0.0126	1.0000	0.0175	−0.1132	0.1451
−0.1132	0.1451	0.0175	0.3750	−0.0323	−0.0323
0.1451	0.0175	−0.1132	−0.0323	0.3750	−0.0323
0.0175	−0.1132	0.1451	−0.0323	−0.0323	0.3750