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INTERNATIONAL INSTITUTE OF ACOUSTIC TECHNOLOGY

Study of finite-difference time-domain analysis on sound fields with porous materials

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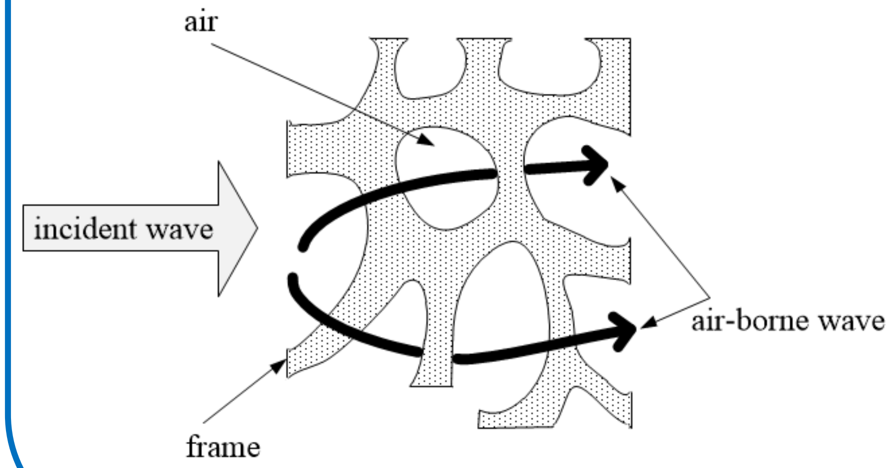
Contents

Rigid frame porous materials



Air-borne wave only →

Equivalent fluid model

$$j\omega p + K_{eq}(\omega)\cdot\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial p}{\partial x} + \rho_{eq}(\omega)\cdot j\omega u = 0$$

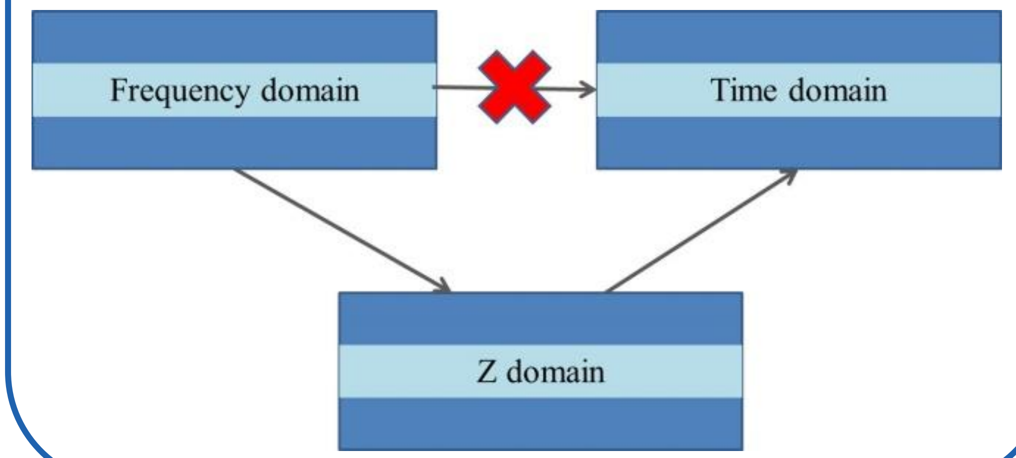
Frequency dependent parameters:

$$K_{eq}(\omega) \qquad \rho_{eq}(\omega)$$

Difficulty in FDTD analysis: Time domain simulation treating frequency dependent parameters

1 EF-FDTD algorithm

$$j\omega p + K_{eq}(\omega)\cdot\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial p}{\partial x} + \rho_{eq}(\omega)\cdot j\omega u = 0$$

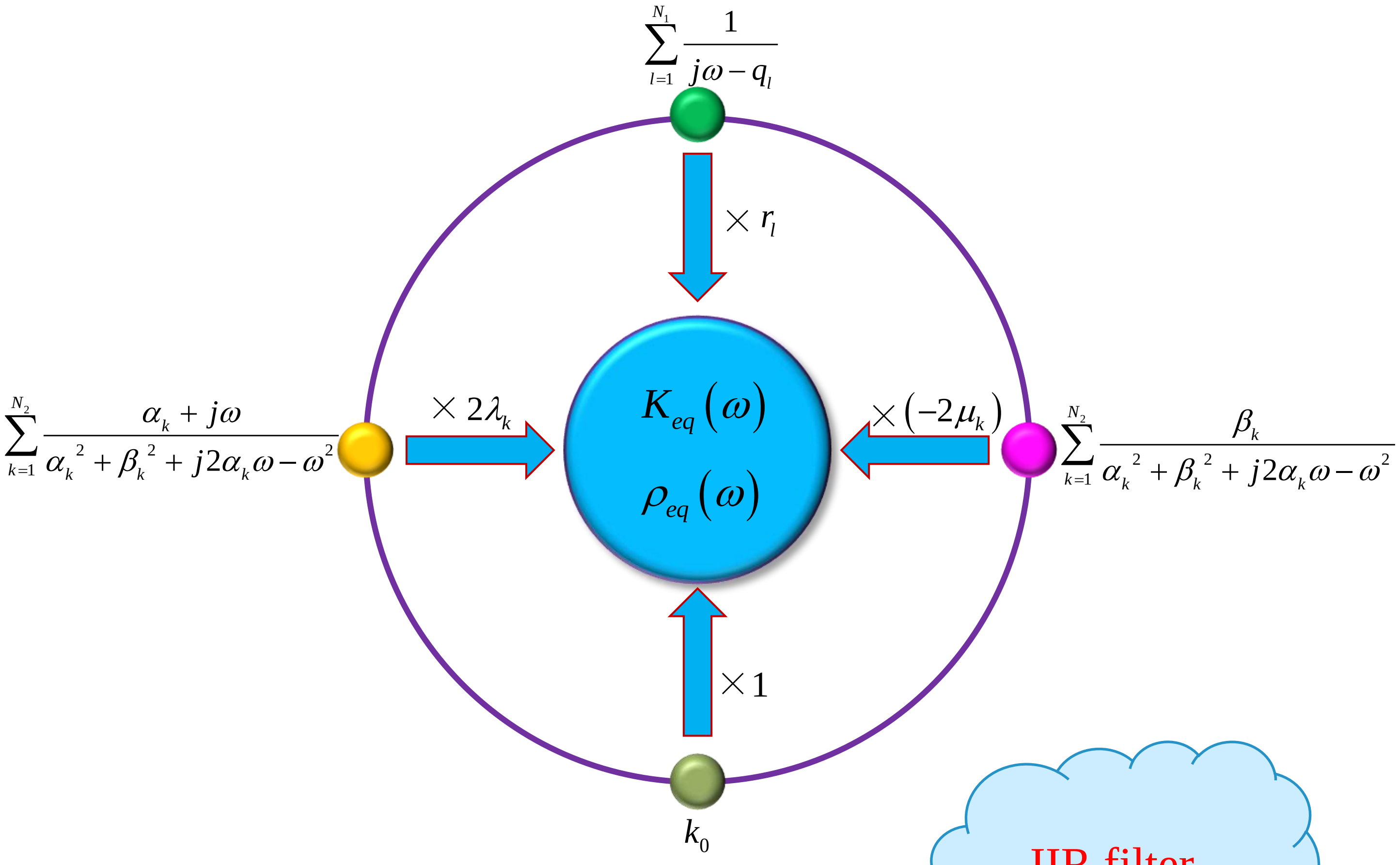


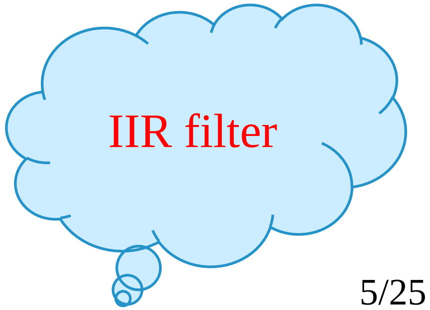
① Transform the whole wave equations to the Z domain
② Treat the wave equations in the Z domain
③ Transform the equations back to the time domain

Time domain	Z domain
$\cdot$	$\cdot$
$+$	$+$
$j\omega p$	$\frac{1-z^{-1}}{\Delta t} P_z(i)$
$j\omega u$	$\frac{1-z^{-1}}{\Delta t} U_z\left(i+\frac{1}{2}\right)$
$\frac{\partial u}{\partial x}$	$\frac{1}{\Delta x}\cdot\left[U_z\left(i+\frac{1}{2}\right)-U_z\left(i-\frac{1}{2}\right)\right]$
$\frac{\partial p}{\partial x}$	$\frac{1}{\Delta x}\cdot\left[P_z(i+1)-P_z(i)\right]$
$K_{eq}(\omega)$	
$\rho_{eq}(\omega)$	

- IIR filter design for $\rho_{eq}(\omega)$ and $K_{eq}(\omega)$

Frequency domain		Z domain
$\frac{1}{j\omega+\alpha}$	⇢	$\frac{1}{1-z^{-1}e^{-\alpha\Delta t}}$
$\frac{\beta}{(\alpha^2+\beta^2)+j2\alpha\omega-\omega^2}$	⇢	$\frac{e^{-\alpha\Delta t}\cdot\sin(\beta\Delta t)\cdot z^{-1}}{1-2e^{-\alpha\Delta t}\cos(\beta\Delta t)\cdot z^{-1}+e^{-2\alpha\Delta t}\cdot z^{-2}}$
$\frac{\alpha+j\omega}{(\alpha^2+\beta^2)+j2\alpha\omega-\omega^2}$	⇢	$\frac{1-e^{-\alpha\Delta t}\cdot\cos(\beta\Delta t)\cdot z^{-1}}{1-2e^{-\alpha\Delta t}\cos(\beta\Delta t)\cdot z^{-1}+e^{-2\alpha\Delta t}\cdot z^{-2}}$
constant in frequency domain	⇢	constant in Z domain





- The equation of the continuity

$$\frac{1-z^{-1}}{\Delta t}P_z(i)+\left[k_0+H_1(z)\Delta t+H_2(z)\Delta t\right]\cdot\frac{1}{\Delta x}\cdot\left[U_z\left(i+\frac{1}{2}\right)-U_z\left(i-\frac{1}{2}\right)\right]=0$$

$$\frac{1-z^{-1}}{\Delta t}P_z(i)+\left[k_0+\sum_{l=1}^{N_1}\left(r_l\Delta t+\frac{a_l\Delta t\cdot z^{-1}}{1-b_l\cdot z^{-1}}\right)+\sum_{k=1}^{N_2}\left(2\lambda_k\Delta t+\frac{X1_k\Delta t\cdot z^{-1}-X2_k\Delta t\cdot z^{-2}}{1-X3_k\cdot z^{-1}+X4_k\cdot z^{-2}}\right)\right]\cdot$$
$$\frac{1}{\Delta x}\cdot\left[U_z\left(i+\frac{1}{2}\right)-U_z\left(i-\frac{1}{2}\right)\right]=0$$

$$D(i)=\frac{1}{\Delta x}\left[U_z\left(i+\frac{1}{2}\right)-U_z\left(i-\frac{1}{2}\right)\right]$$

$$I_l(i)=\frac{D(i)}{1-b_l\cdot z^{-1}},\quad l=1,2\cdots N_1$$

$$Q_k(i)=\frac{D(i)}{1-X3_k\cdot z^{-1}+X4_k\cdot z^{-2}},\quad k=1,2\cdots N_2$$

$$P_z(i)=z^{-1}P_z(i)-K_e\Delta t\cdot D(i)-\Delta t^2\sum_{l=1}^{N_1}a_l\cdot z^{-1}I_l(i)-\Delta t^2\sum_{k=1}^{N_2}X1_k\cdot z^{-1}Q_k(i)-X2_k\cdot z^{-2}Q_k(i)$$

- The equation of the motion

$$\frac{1-z^{-1}}{\Delta t}\cdot\left[k_0'+H_1'(z)\Delta t+H_2'(z)\Delta t\right]\cdot U_z\left(i+\frac{1}{2}\right)+\frac{1}{\Delta x}\cdot\left[P_z(i+1)-P_z(i)\right]=0$$

$$\frac{1-z^{-1}}{\Delta t}\cdot\left[k_0'+\sum_{l=1}^{N_1'}\left(r_l'\Delta t+\frac{a_l'\Delta t\cdot z^{-1}}{1-b_l'\cdot z^{-1}}\right)+\sum_{k=1}^{N_2'}\left(2\lambda_k'\Delta t+\frac{X1_k'\Delta t\cdot z^{-1}-X2_k'\Delta t\cdot z^{-2}}{1-X3_k'\cdot z^{-1}+X4_k'\cdot z^{-2}}\right)\right]\cdot$$
$$U_z\left(i+\frac{1}{2}\right)+\frac{1}{\Delta x}\cdot\left[P_z(i+1)-P_z(i)\right]=0$$

$$M_l\left(i+\frac{1}{2}\right)=\frac{1}{1-b_l'\cdot z^{-1}}\cdot U_z\left(i+\frac{1}{2}\right),\quad l=1,2\cdots N_1'$$

$$R_k\left(i+\frac{1}{2}\right)=\frac{1}{1-X3_k'\cdot z^{-1}+X4_k'\cdot z^{-2}}\cdot U_z\left(i+\frac{1}{2}\right),\quad k=1,2\cdots N_2'$$

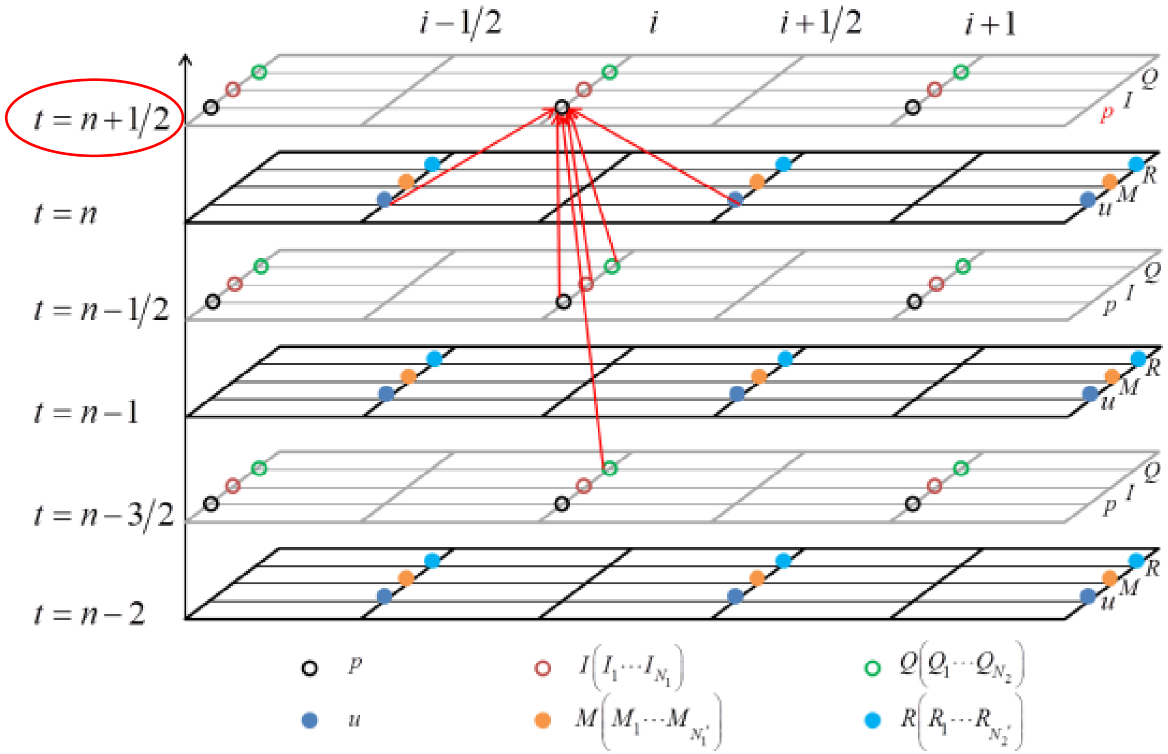
$$U_z\left(i+\frac{1}{2}\right)=z^{-1}U_z\left(i+\frac{1}{2}\right)-\frac{\Delta t}{\rho_e\Delta x}\left[P_z(i+1)-P_z(i)\right]-\frac{\Delta t}{\rho_e}\sum_{l=1}^{N_1'}a_l'\left[z^{-1}M_l\left(i+\frac{1}{2}\right)-z^{-2}M_l\left(i+\frac{1}{2}\right)\right]$$
$$-\frac{\Delta t}{\rho_e}\sum_{k=1}^{N_2'}X1_k'\left[z^{-1}R_k\left(i+\frac{1}{2}\right)-z^{-2}R_k\left(i+\frac{1}{2}\right)\right]+\frac{\Delta t}{\rho_e}\sum_{k=1}^{N_2'}X2_k'\left[z^{-2}R_k\left(i+\frac{1}{2}\right)-z^{-3}R_k\left(i+\frac{1}{2}\right)\right]$$

1 EF-FDTD algorithm

■ EF-FDTD step 1: change of p

$$D(i)=\frac{1}{\Delta x}\left[u^{n}(i+1/2)-u^{n}(i-1/2)\right]$$

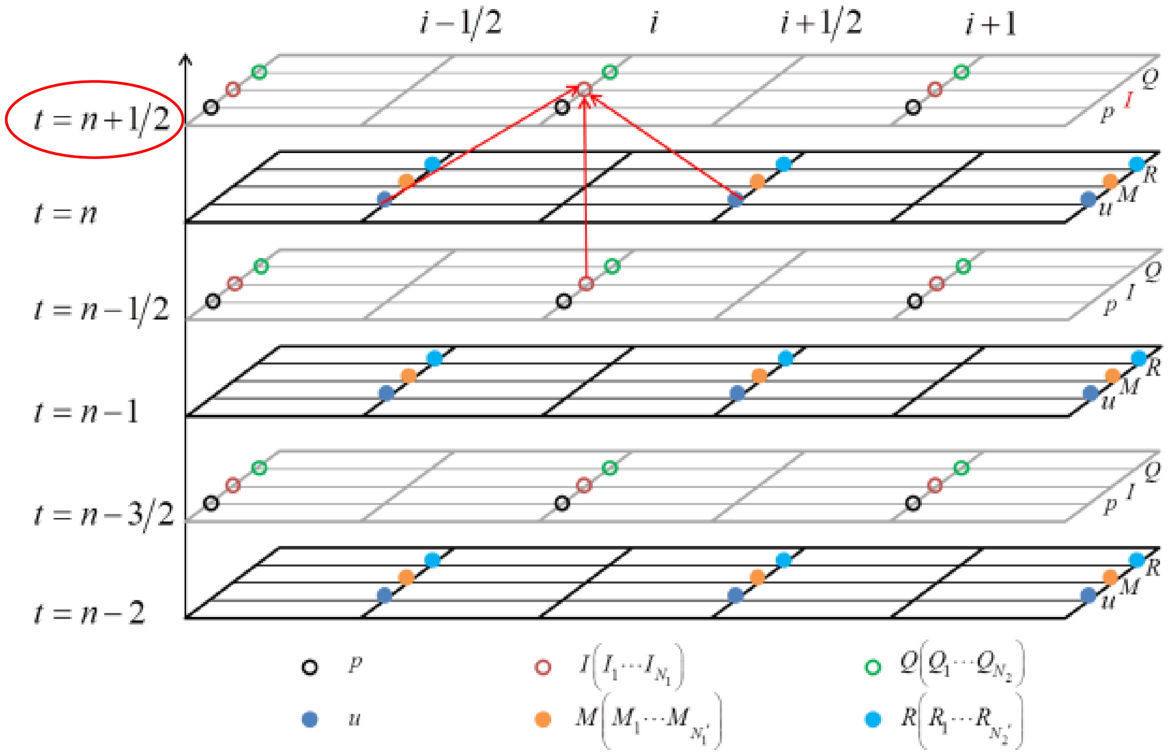
$$p^{n+\frac{1}{2}}(i)=p^{n-\frac{1}{2}}(i)-K_e\Delta t\cdot D(i)-\Delta t^2\sum_{l=1}^{N_1}a_l\cdot I_l^{n-\frac{1}{2}}(i)-\Delta t^2\sum_{k=1}^{N_2}X1_k\cdot Q_k^{n-\frac{1}{2}}(i)-X2_k\cdot Q_k^{n-\frac{3}{2}}(i)$$



1 EF-FDTD algorithm

- EF-FDTD step 2: change of I

$$I_l^{n+\frac{1}{2}}(i) = b_l \cdot I_l^{n-\frac{1}{2}}(i) + D(i), \quad l = 1,2\cdots N_1$$



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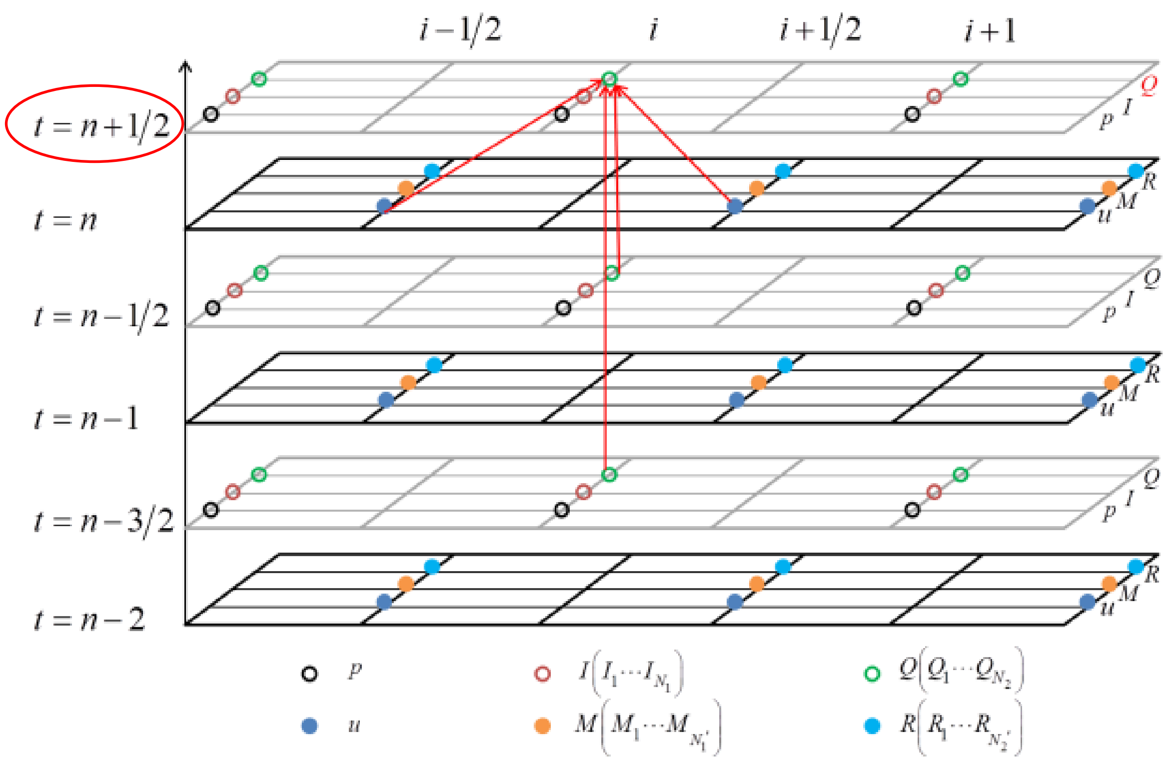
- EF-FDTD step 3: change of Q

$$Q_k^{n+\frac{1}{2}}(i) = X3_k \cdot Q_k^{n-\frac{1}{2}}(i) - X4_k \cdot Q_k^{n-\frac{3}{2}}(i) + D(i)$$
$$k = 1, 2 \cdots N_2$$



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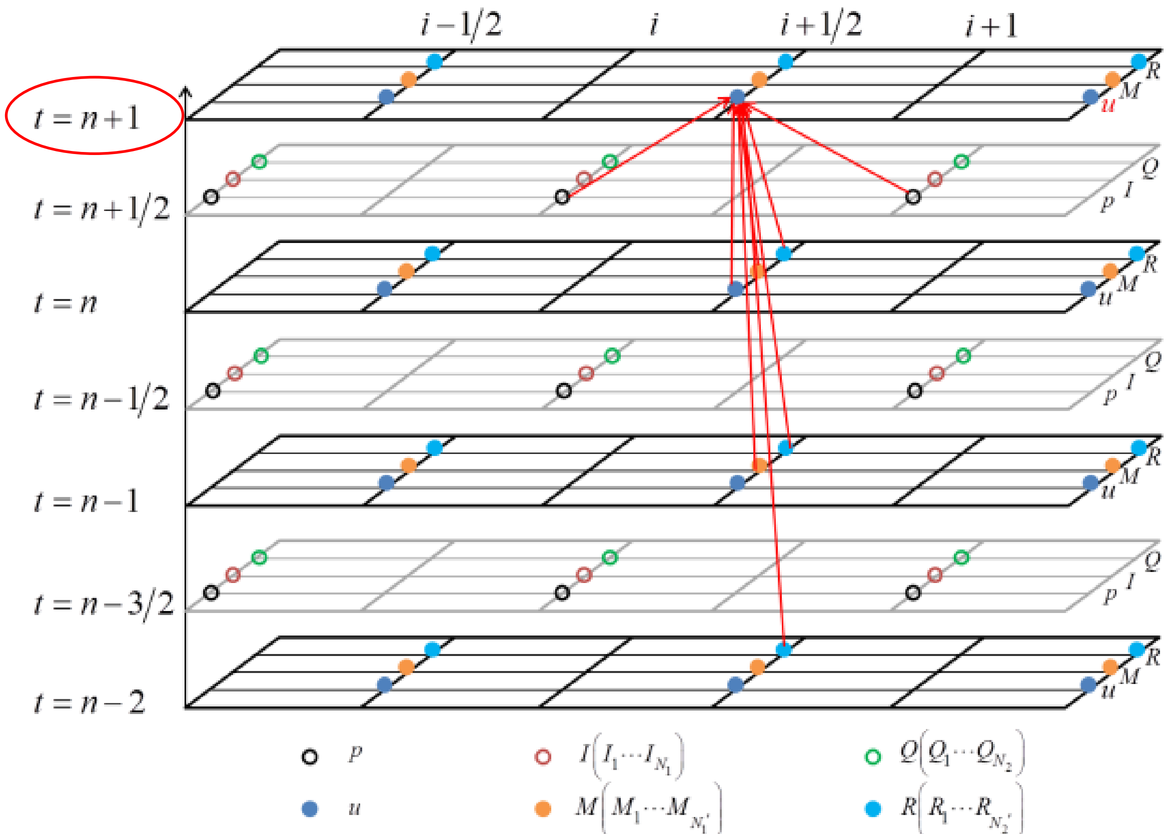
- EF-FDTD step 4: change of u

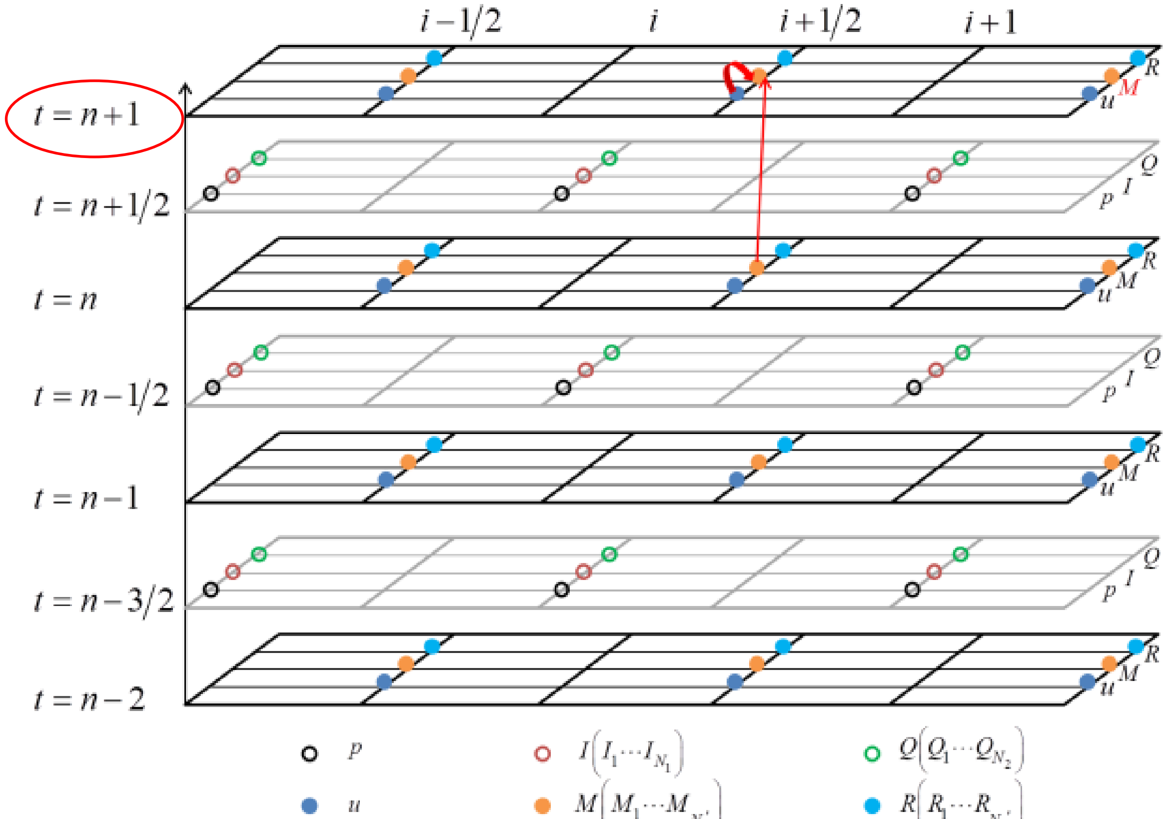
$$u^{n+1}\left(i+\frac{1}{2}\right)=u^{n}\left(i+\frac{1}{2}\right)-\frac{\Delta t}{\rho_e\Delta x}\left[p^{n+\frac{1}{2}}(i+1)-p^{n+\frac{1}{2}}(i)\right]-\frac{\Delta t}{\rho_e}\sum_{l=1}^{N_1'}a_l'\cdot\left[M_l^{\,n}\left(i+\frac{1}{2}\right)-M_l^{\,n-1}\left(i+\frac{1}{2}\right)\right]$$
$$-\frac{\Delta t}{\rho_e}\sum_{k=1}^{N_2'}X1_k'\cdot\left[R_k^{\,n}\left(i+\frac{1}{2}\right)-R_k^{\,n-1}\left(i+\frac{1}{2}\right)\right]+\frac{\Delta t}{\rho_e}\sum_{k=1}^{N_2'}X2_k'\cdot\left[R_k^{\,n-1}\left(i+\frac{1}{2}\right)-R_k^{\,n-2}\left(i+\frac{1}{2}\right)\right]$$



EF-FDTD step 5: change of M

$$M_l^{n+1}\left(i+\frac{1}{2}\right)=u^{n+1}\left(i+\frac{1}{2}\right)+b_l'\cdot M_l^{n}\left(i+\frac{1}{2}\right)$$

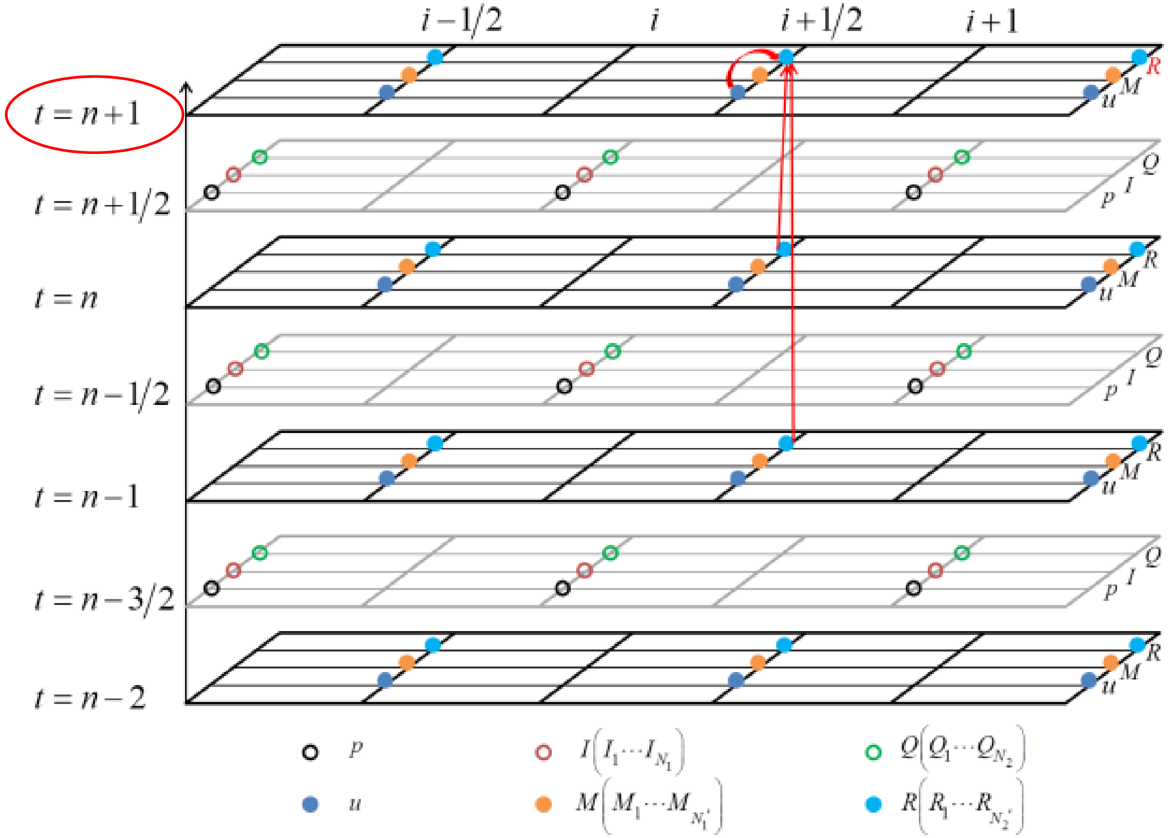
$$l=1,2\cdots N_1'$$



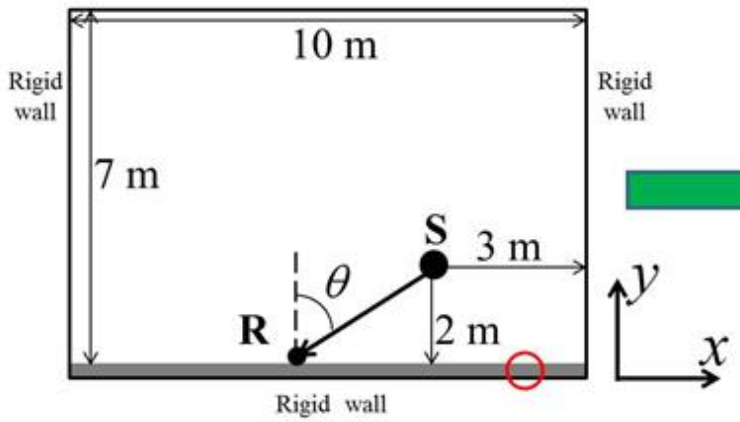
- EF-FDTD step 6: change of R

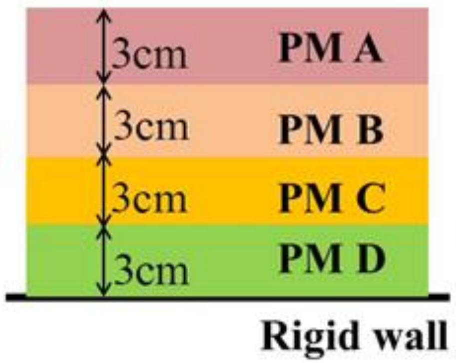
$$R_k^{n+1}\left(i+\frac{1}{2}\right)=u^{n+1}\left(i+\frac{1}{2}\right)+X3_k'\cdot R_k^{n}\left(i+\frac{1}{2}\right)-X4_k'\cdot R_k^{n-1}\left(i+\frac{1}{2}\right)$$

$$k=1,2\cdots N_2'$$



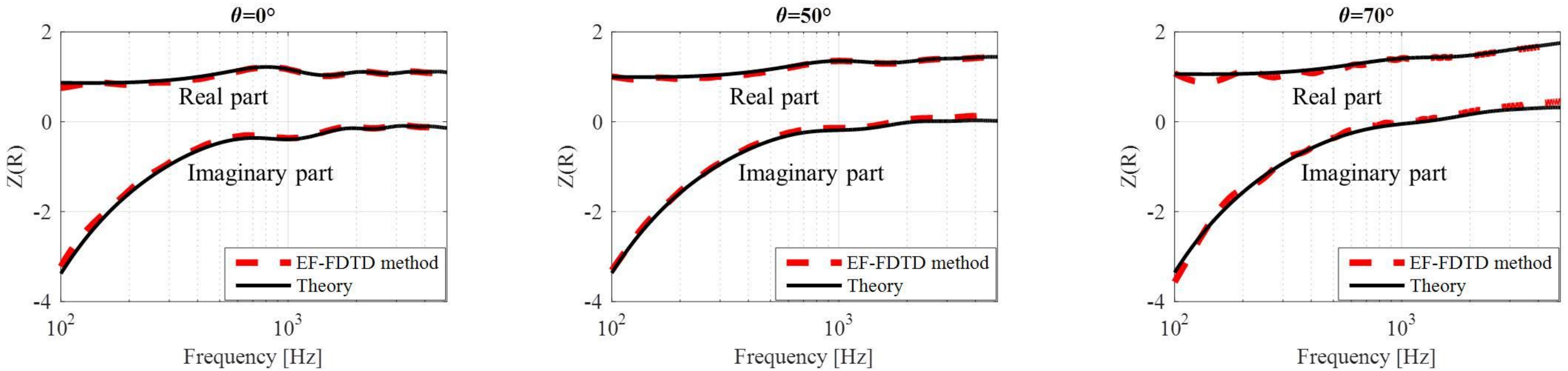
■ Simulation





- 4 layers：
  - PM A：Flow resistivity 5000 $Nm^{-4}s$
  - PM B：Flow resistivity 10000 $Nm^{-4}s$
  - PM C：Flow resistivity 15000 $Nm^{-4}s$
  - PM D：Flow resistivity 20000 $Nm^{-4}s$

■ The calculated and theoretical surface impedance at different incident angle



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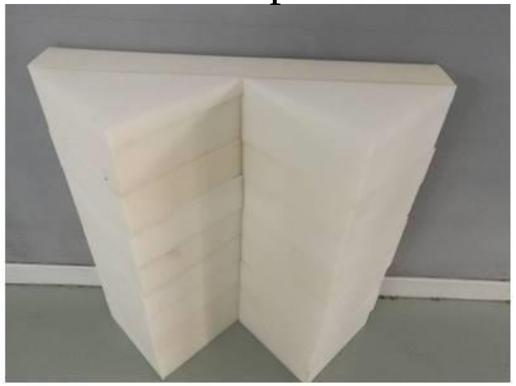
■ Developing a method for predicting the sound absorption coefficient of acoustic wedges by using the EF-FDTD algorithm.

Sample A

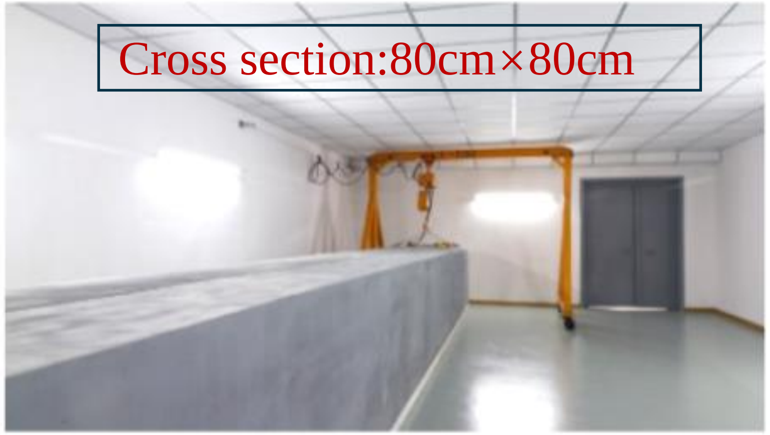
Sample B



Sample C

Sample D

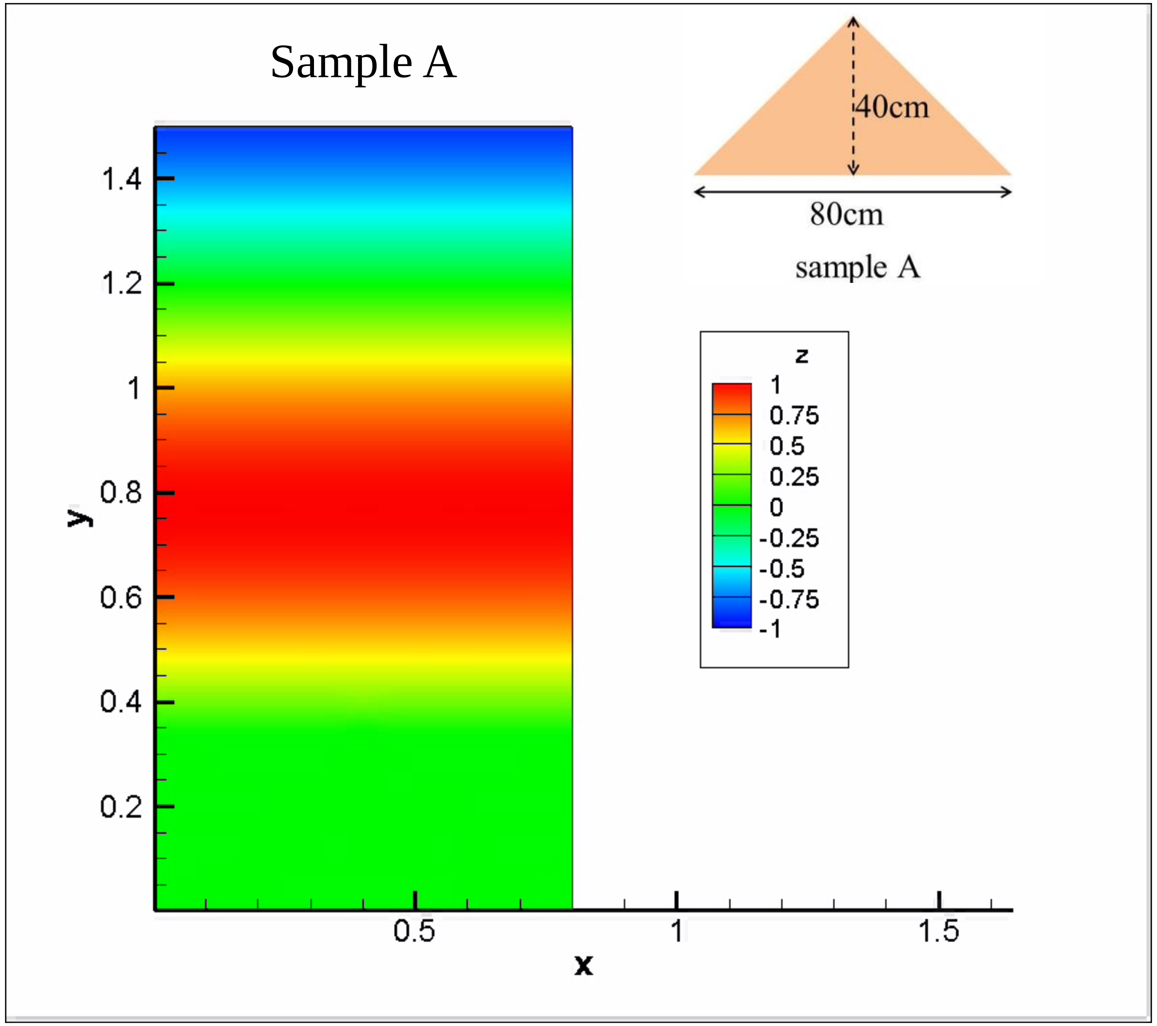
- Measurement in the low frequency impedance tube
- Numerical simulation by using EF-FDTD algorithm

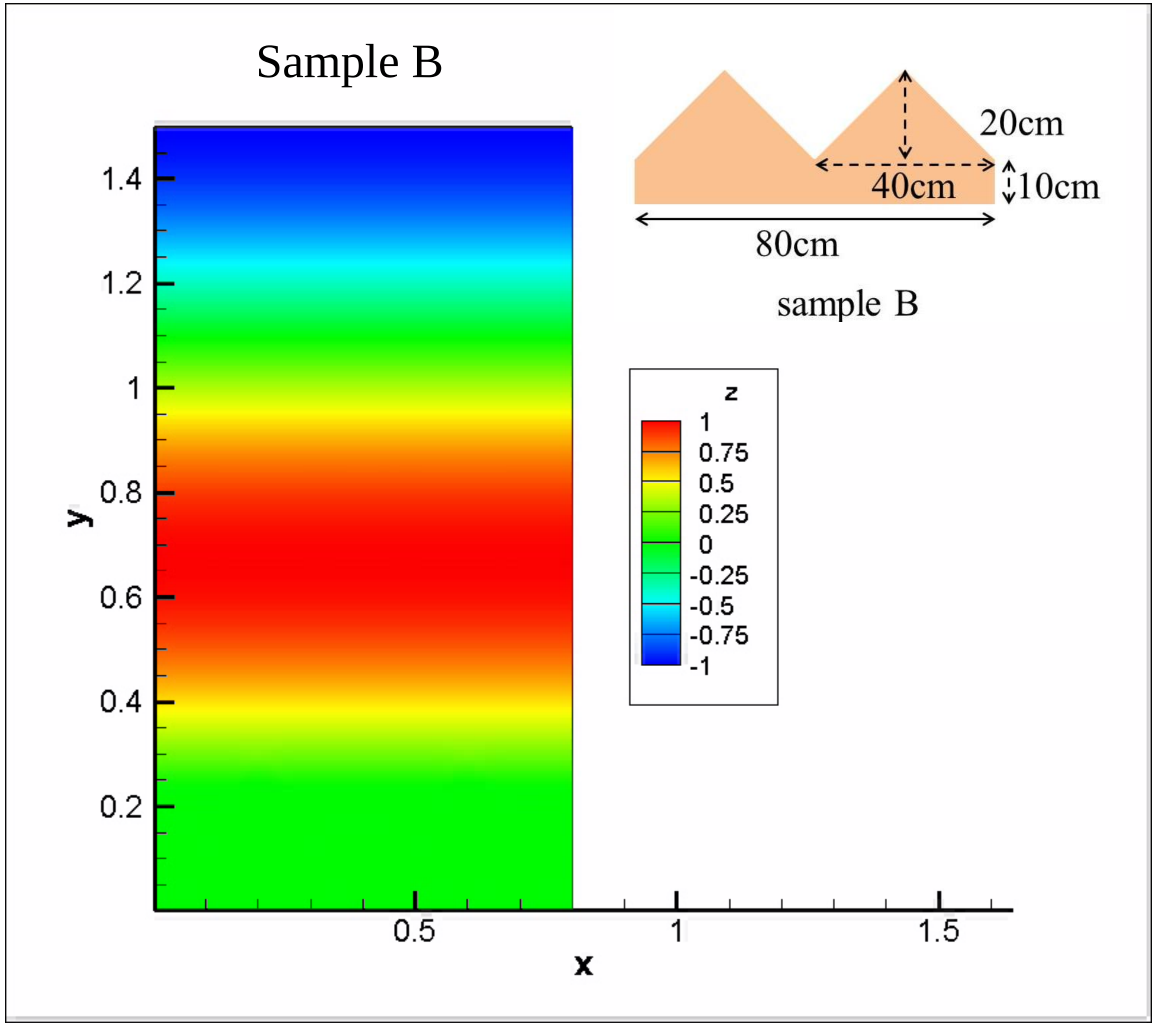


Low frequency impedance tube

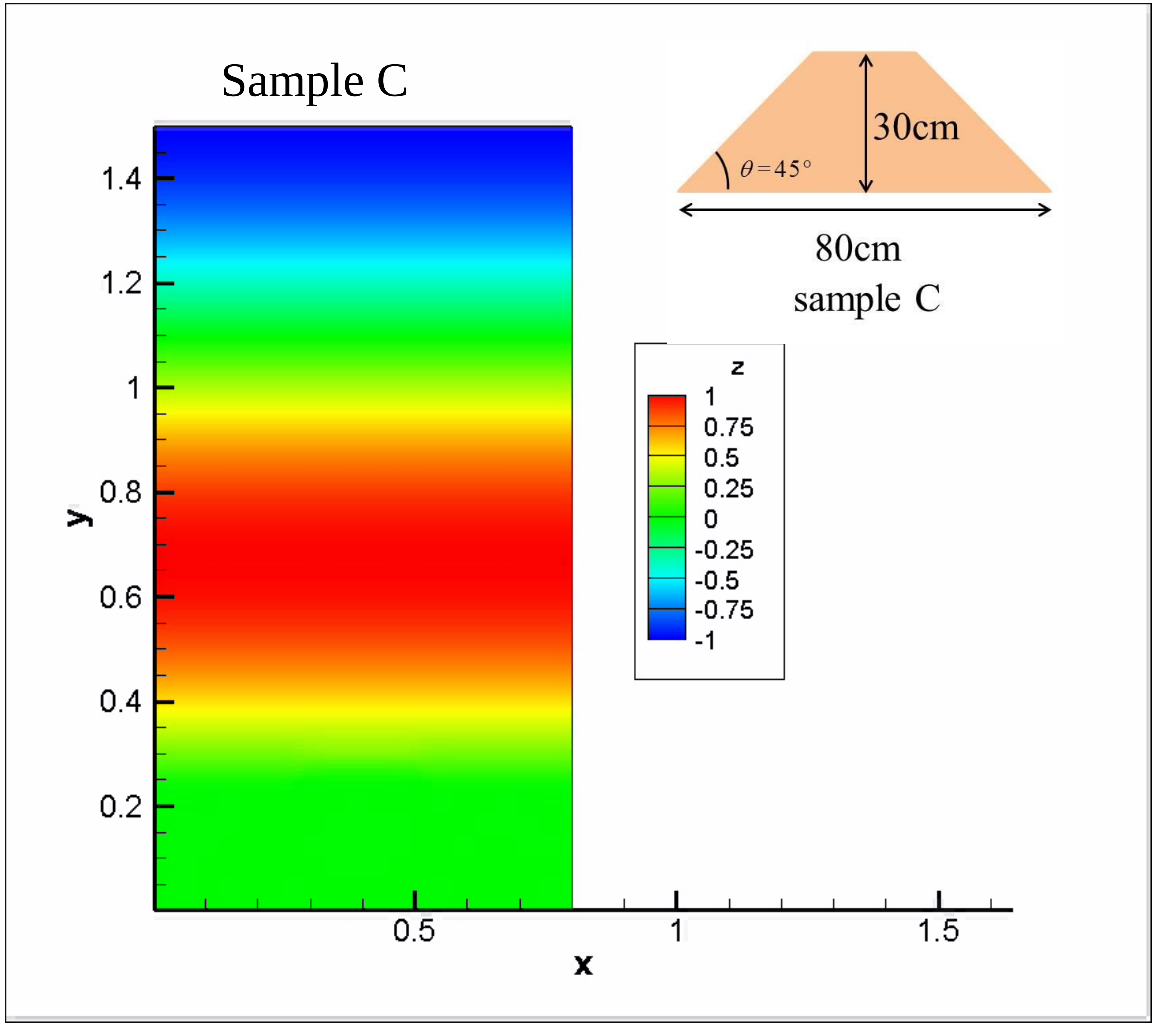
2 Simulation of the acoustic wedges

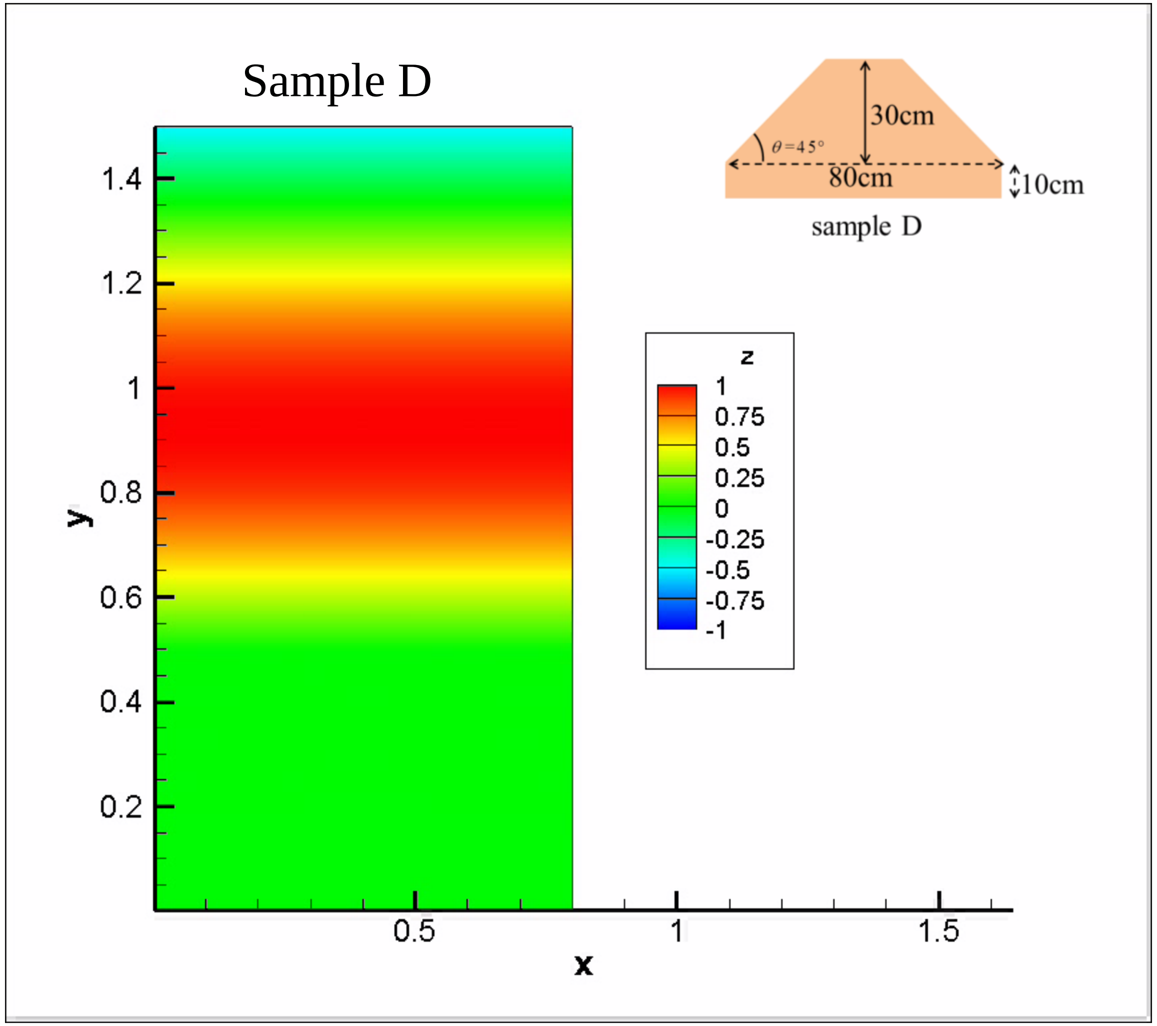
- Simulation of the sound field (200Hz)



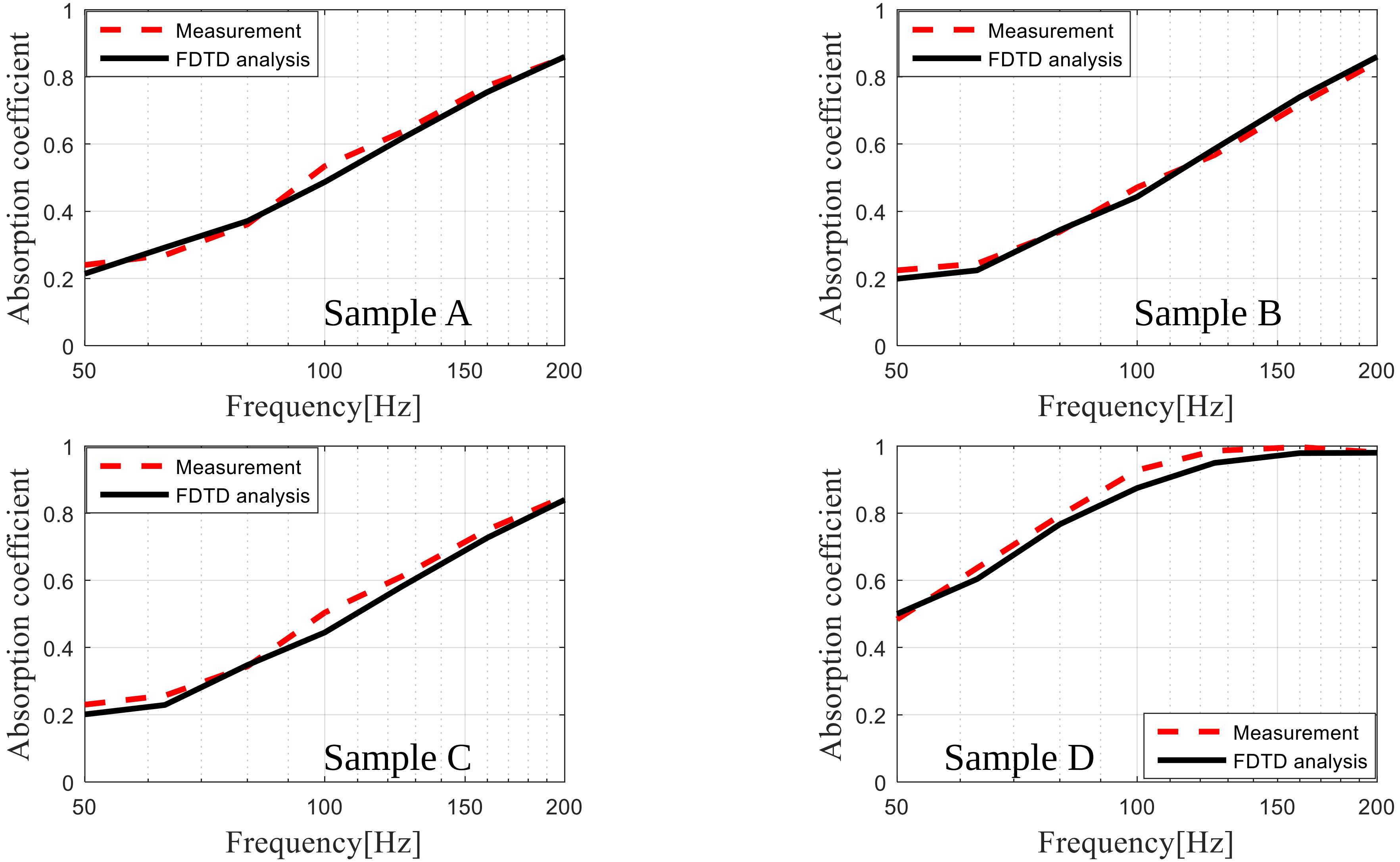


■ Simulation of the sound field (200Hz)



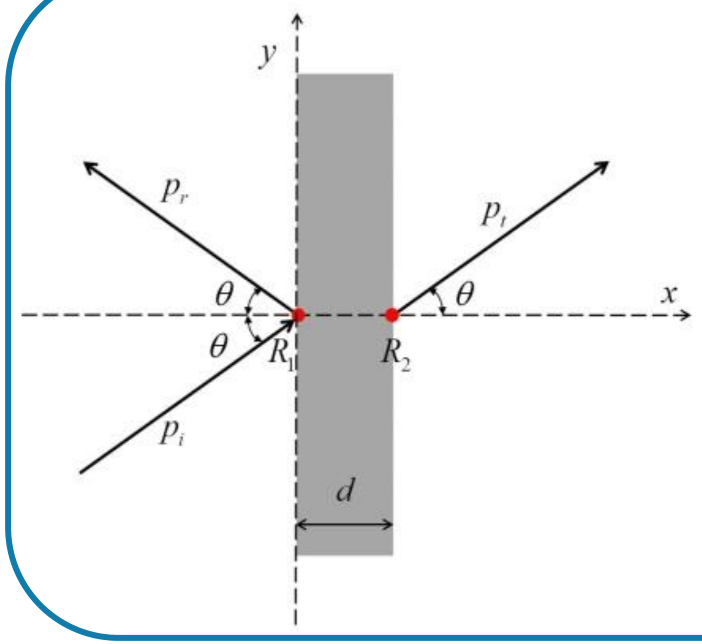


■ Absorption coefficients



Contents

■ From the total sound pressure and the total particle velocity at R1, the incident sound pressure and the incident particle velocity can be separated.



Incident Power

$$P_I = \frac{A^2\cos\theta}{2\rho c} = \frac{1}{2}\left[\left|I_{cx}(x)\right|_{\max} + I_x(x)\right]$$

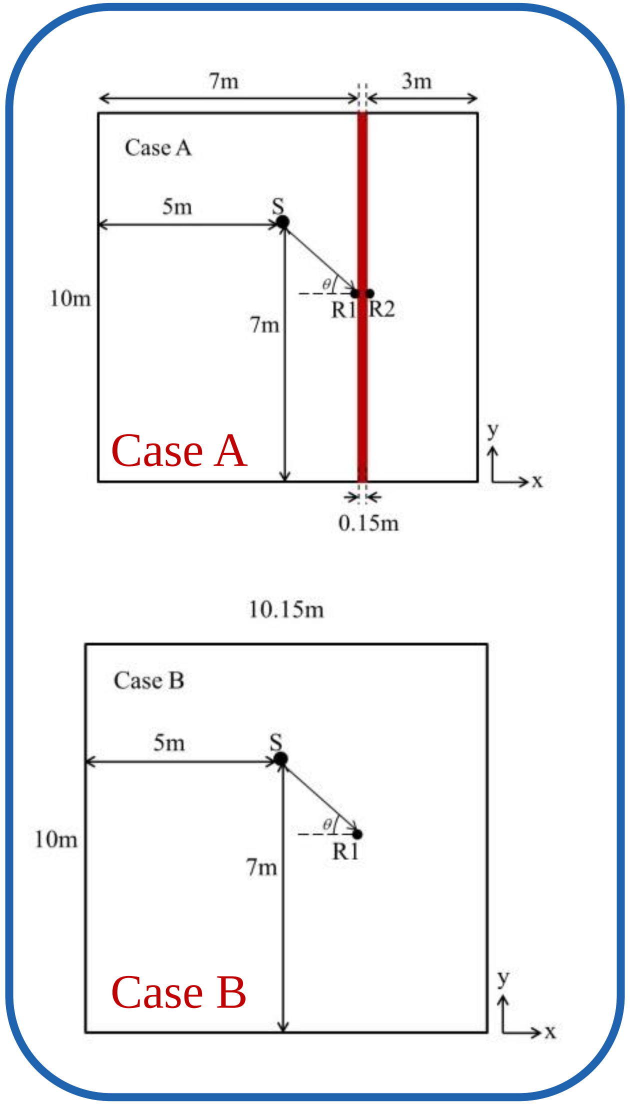
$$\left|I_{cx}(x)\right|_{\max} = \frac{I_x^2(x) + Q_x^2(x) + \left[\bar{p}^2(x)/\rho c\right]^2\cos^2\theta}{2\left[\bar{p}^2(x)/\rho c\right]\cos\theta}$$

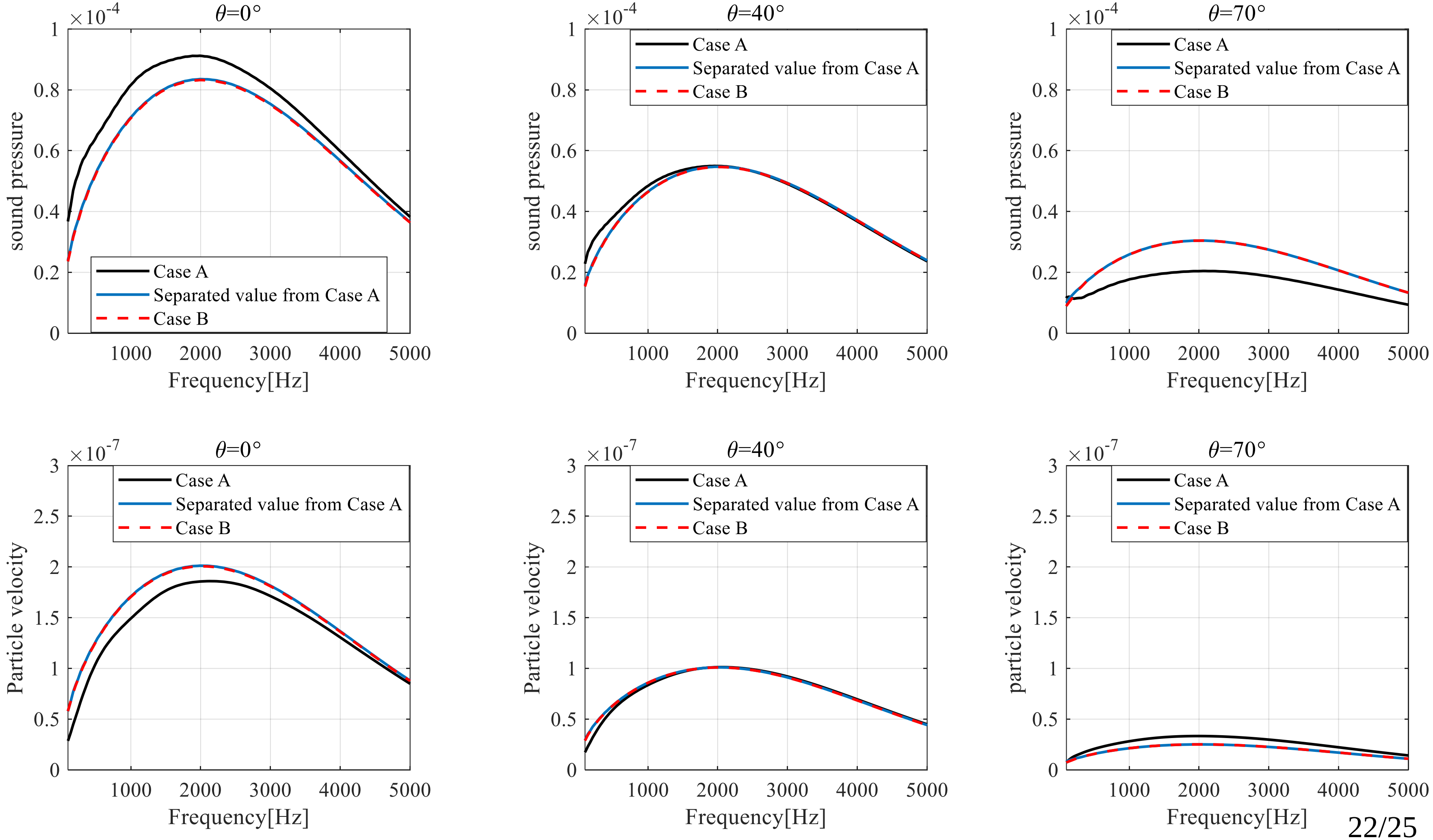
$$\bar{p}^2(x) = \frac{1}{2}p(x)\cdot p^*(x) = \frac{1}{2}\left(1+r^2\right)A^2 + rA^2\cos\left(2kx\cos\theta+\varphi\right)$$

Incident sound pressure： $|p_i| = A = \sqrt{\dfrac{2\rho c P_I}{\cos\theta}}$

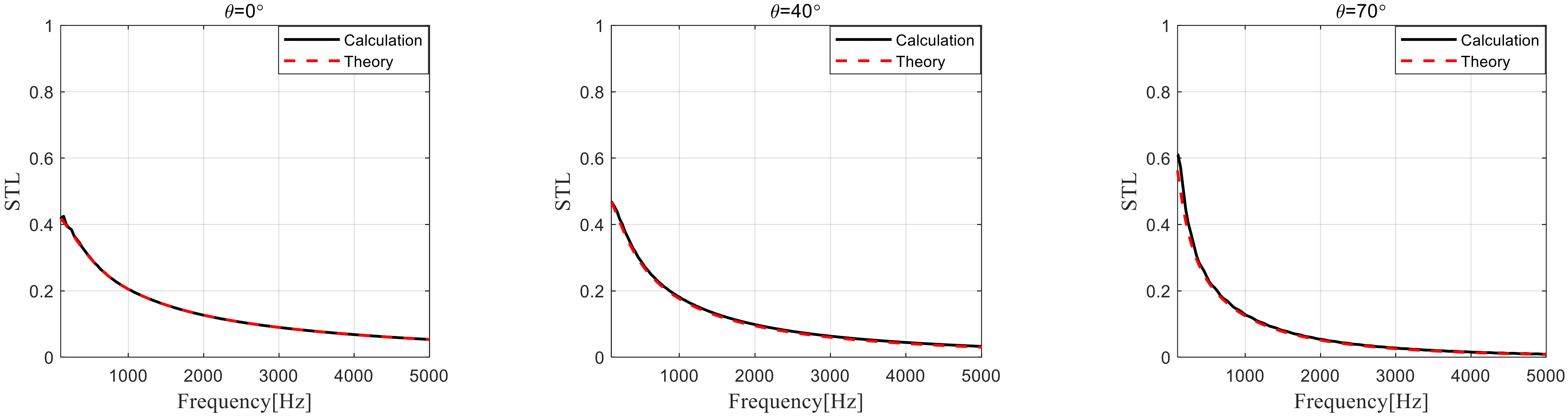
Incident particle velocity: $|u_i| = \dfrac{A\cos\theta}{\rho c} = \sqrt{\dfrac{2P_I\cos\theta}{\rho c}}$

■ Separated incident sound pressure and the incident particle velocity at R1



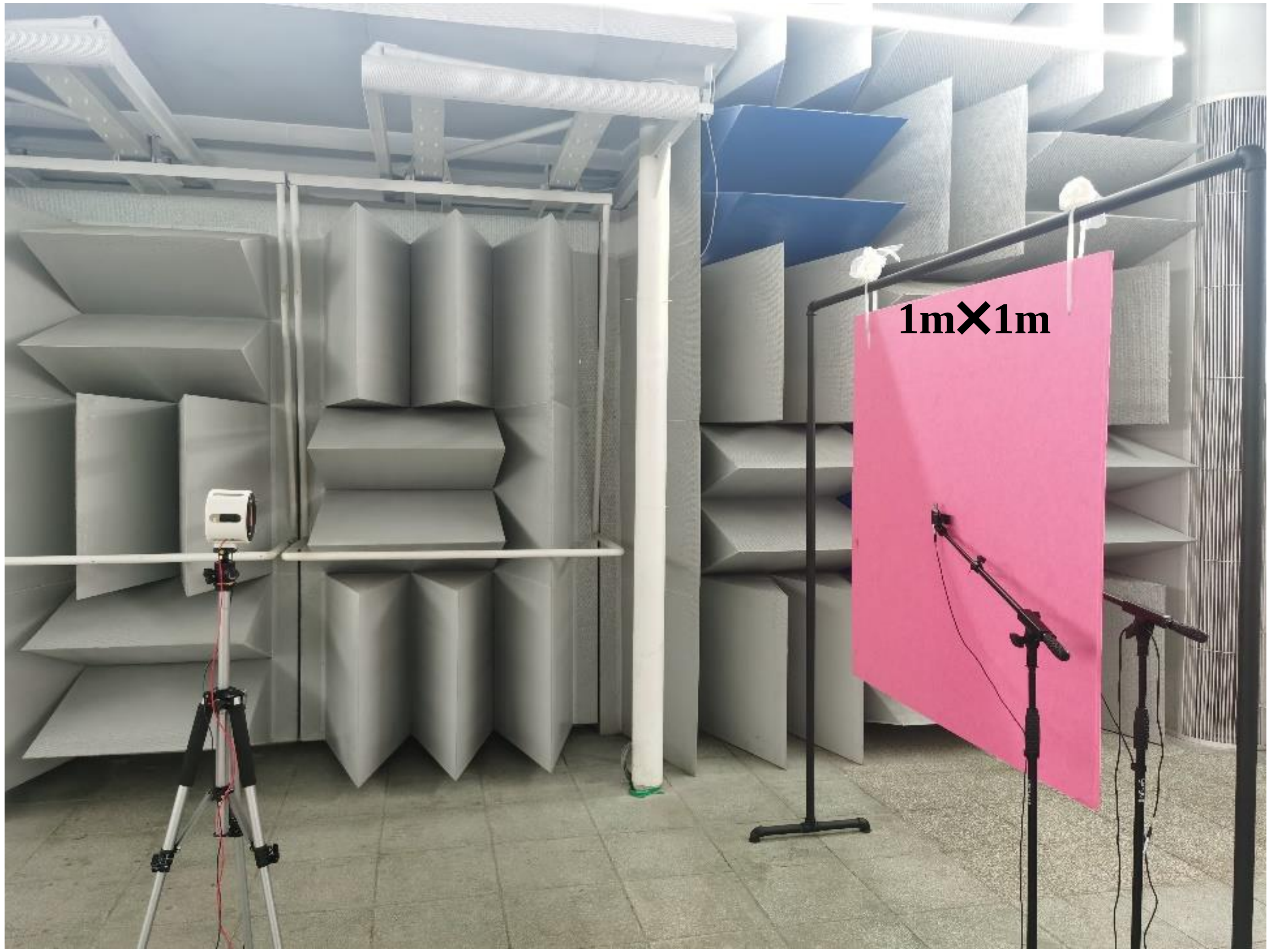


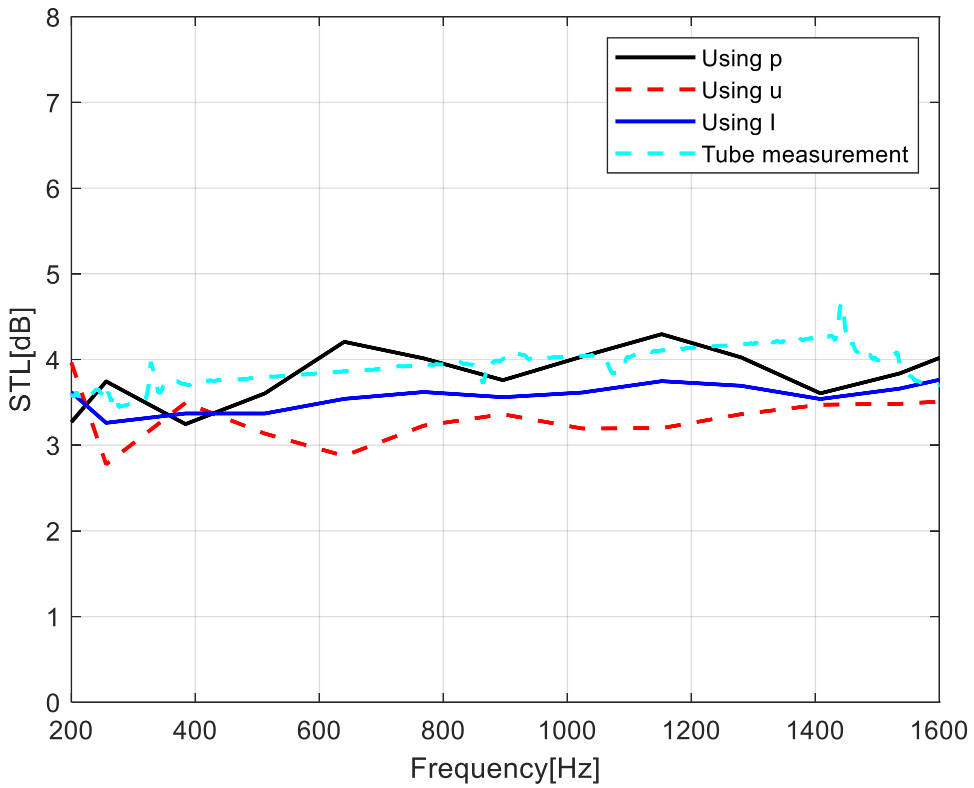
■ Calculated STL compared with the theoretical value



- In-situ measurement for the sound transmission loss
- By using sound pressure-particle velocity probe (pu probe)

■ Measurement in the semi-anechoic room





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THANKS

Abstract

An equivalent fluid model based finite-difference time-domain algorithm is used to simulate the frequency characteristic of porous material with rigid frame. The effective density and the effective bulk modulus are frequency-dependent complex values, which are designed in the form of infinite impulse response filters in the frequency domain. The Z transform theory is used to discretize the frequency-domain wave equations. By using the EF-FDTD algorithm, the sound absorption coefficient of acoustic wedges can be predicted. The transmission loss of porous materials with different incident angles can be simulated, which provided evidence for the feasibility of measuring STL in situ using one particle velocity–pressure sensor and one pressure sensor.