On pulse propagation in poro-visco-elastic felt-like material

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Motivation

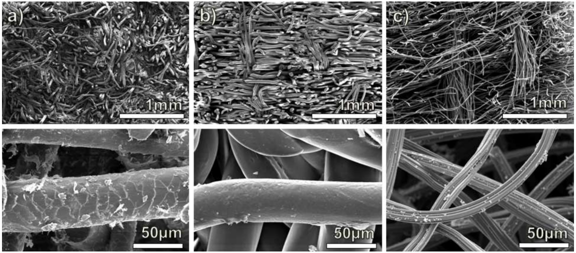


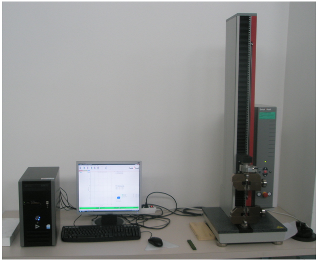
Figure: SEM images[1] of a) wool felt, b) polypropylene felt and c) carbon felt.

- Felt is a non-woven fabric (composite material) produced by wet felting natural or synthetic fibres
- Felt properties are less understood than the properties of the fibers within them
- Strain wave propagation, dispersion and dissipation through felts
- Wide range of applications

[1] J. S. Winzer, *Production and Characterisation of Alumina-Copper Interpenetrating Composites*, VVB Laufersweiler Verlag, Darmstadt, Germany, 2013.

Experimental studies of wool felt and piano hammers[2]

Piano hammer collision dynamics: Static and dynamic testing of **piano hammers** and **felt pads**. Characterising and measuring of properties of wool felt.



[2]A. Stulov, A. Mägi, "Piano hammer testing device," *Proc. Estonian Acad. Sci. Engin.*, **6**(4), pp. 259–267, 2000.

Theoretical studies of wool felt and piano hammers[3,4]

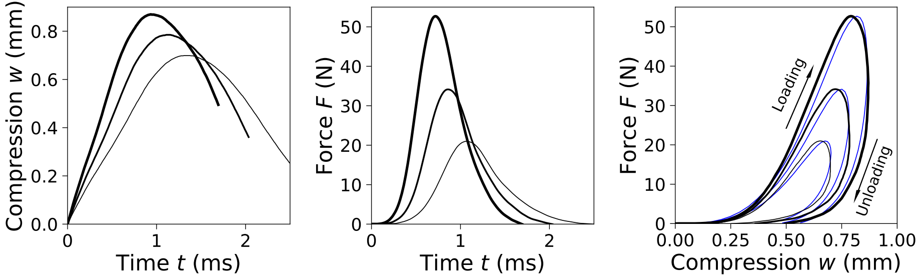


Figure: Typical hammer collisions lasting for t_{c} s. Experimental data (black) vs. fitted model data (blue).

$$F(t) = F_0 \left[w^p(t) - \frac{\gamma}{\tau} \int_0^t w^p(\xi) \exp\left(\frac{\xi - t}{\tau}\right) \mathrm{d}\xi \right], \tag{1}$$

where γ is the hereditary amplitude, τ is the relaxation time, p is the compliance exponent, F_0 is the instantaneous stiffness.

[3] A. Stulov, "Hysteretic model of the grand piano hammer felt," *J. Acoust. Soc. Am.*, **97**(4), pp. 2577–2585, 1995.

[4] A. Stulov, "Dynamic behavior and mechanical features of wool felt," *Acta Mech.*, **169**, pp. 13–21, 2004.

Wool felt model

A **hereditary visco-elastic continuum** can't be described by the static Young's modulus E. Instead, the modulus is replaced by a time-dependant operator in the form $E\left[1+\mathcal{R}(t)*\right]$ here

$$\mathcal{R}(t)=\frac{\gamma}{\tau_0}\exp\left(-\frac{t}{\tau_0}\right), \qquad 0\leqslant\gamma<1, \tag{2}$$

where γ is the hereditary amplitude and τ_0 is the relaxation time.
Constitutive relation of the felt is proposed in the following form[5,6]:

$$\sigma(\varepsilon)=E_{\rm d}\left[\varepsilon^p(t)-\mathcal{R}(t)*\varepsilon^p(t)\right]= \\ =E_{\rm d}\left[\varepsilon^p(t)-\frac{\gamma}{\tau_0}\int_0^t\varepsilon^p(\xi)\exp\left(\frac{\xi-t}{\tau_0}\right)\mathrm{d}\xi\right], \tag{3}$$

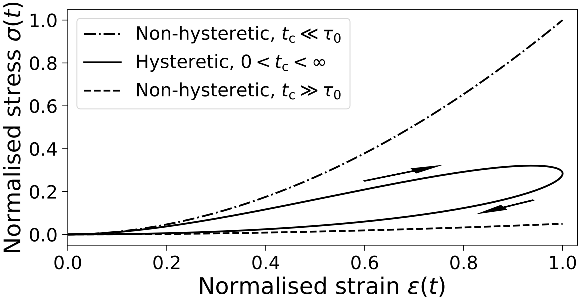
here $E_{\rm d}$ is the dynamic Young's modulus, p is the compliance exponent.

[5] D. Kartofelev, A. Stulov, "Propagation of deformation waves in wool felt," *Acta Mech*, **225**, pp. 3103–3113, 2014.

[6] D. Kartofelev, A. Stulov, "Wave propagation and dispersion in microstructured wool felt," *Wave Motion*, **57**, pp. 23–33, 2015.

Analysis of constitutive relation (3)

$$\sigma(\varepsilon) = E_\mathrm{d}\left[\varepsilon^p(t) - \frac{\gamma}{\tau_0}\int_0^t \varepsilon^p(\xi)\exp\left(\frac{\xi - t}{\tau_0}\right)\mathrm{d}\xi\right]$$



Fast loading cycle, $t_\mathrm{c} \ll \tau_0$:

$$\sigma(\varepsilon) = E_\mathrm{d}\varepsilon^p(t), \quad (4)$$

where E_d is the dynamic Young's modulus.

Slow loading cycle, $t_\mathrm{c} \gg \tau_0$:

$$\sigma(\varepsilon) = E_\mathrm{s}\varepsilon^p(t), \quad (5)$$

where $E_\mathrm{s} = E_\mathrm{d}(1 - \gamma)$ is the static Young's modulus.

Model equation

The felt model equation is derived from 1-D equation of motion

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x}, \tag{6}$$

where ρ is the density and $u(x,t)$ is the displacement. Substitution of constitutive relation (3) into (6) and eliminating of the integral term leads to the following model equation:

$$\rho \frac{\partial^2 u}{\partial t^2} + \rho \tau_0 \frac{\partial^3 u}{\partial t^3} - E_\mathrm{d} \left\{ (1-\gamma) \frac{\partial}{\partial x} \left[\left(\frac{\partial u}{\partial x} \right)^p \right] + \tau_0 \frac{\partial^2}{\partial x \partial t} \left[\left(\frac{\partial u}{\partial x} \right)^p \right] \right\} = 0. \tag{7}$$

In the case of **wool felt** used in certain piano hammers, the values of physical material constants are: $E_\mathrm{s} = 0.6$ MPa, $\rho = 10^3$ kg/m^3, $\underline{\gamma = 0.95}$, $\tau_0 = 10\ \mu$s, $1.0 < p < 3.6$. Dependent parameter values: $E_\mathrm{d} = 15$ MPa, $c_\mathrm{s} = 25$ m/s, $c_\mathrm{d} = 125$ m/s.

Model equation: Dimensionless form

Dimensionless variables are introduced as follows:

$$u \Leftarrow \frac{u}{L}, \qquad x \Leftarrow \frac{x}{L}, \qquad t \Leftarrow \frac{t}{T}, \tag{8}$$

where

$$T = \tau_0/\delta, \quad L = c_\mathrm{d} T \sqrt{\delta}, \quad \delta = 1 - \gamma, \quad c_\mathrm{d} = \sqrt{E_\mathrm{d}/\rho}, \quad c_\mathrm{s} = c_\mathrm{d}\sqrt{\delta}. \tag{9}$$

Thus, the dimensionless form of model equation (7) in terms of strain variable $\varepsilon = \partial u/\partial x$ takes the form:

$$\boxed{\varepsilon_{tt} = (\varepsilon^p)_{xx} + (\varepsilon^p)_{xxt} - \delta \varepsilon_{ttt},} \tag{10}$$

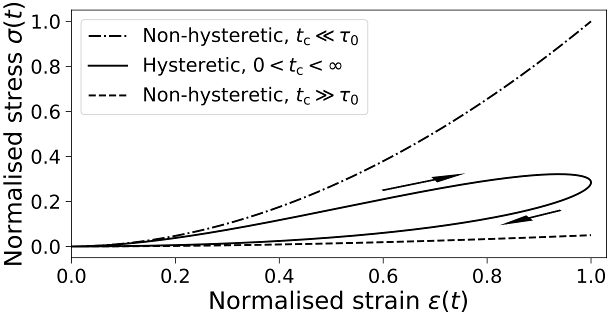
where p is the material compliance exponent and $\delta \in (0, 1]$ is related to all hereditary and dissipative properties. If $\delta \to 0$, i.e., $\gamma \to 1$, then the hereditary properties approach maximum levels and the dissipative properties approach minimum levels.

In the case of physical constant values shown on previous slide:

$L = 6.25$ mm and $T = 0.25$ ms.

Non-hysteretic model equations

The limit cases for **rapid** and **slow** material loading cycles.



Fast loading cycle, $t_\mathrm{c} \ll \tau_0$:

$$\sigma(\varepsilon) = E_\mathrm{d}\varepsilon^p(t).$$

Model equation in dimensionless strain variable:

$$\boxed{\varepsilon_{tt} = (\varepsilon^p)_{xx}.} \qquad (11)$$

Slow loading cycle, $t_\mathrm{c} \gg \tau_0$:

$$\sigma(\varepsilon) = E_\mathrm{s}\varepsilon^p(t).$$

Model equation in dimensionless strain variable:

$$\boxed{\varepsilon_{tt} = \delta^2(\varepsilon^p)_{xx}.} \qquad (12)$$

Dispersion analysis of model equation

The solution to model equation (10) can be assumed in the form

$$\varepsilon(x,t) \propto \mathrm{e}^{\mathrm{i}(kx-\Omega t)}. \tag{13}$$

The **characteristic equation** that corresponds to **linearised** model equation (10) has the following form:

$$\boxed{k^2 - \Omega^2 - \mathrm{i}k^2\Omega + \mathrm{i}\delta\Omega^3 = 0.} \tag{14}$$

In general case angular frequency $\Omega(k)$ is a complex quantity

$$\Omega(k) = \omega(k) + \mathrm{i}\mu(k). \tag{15}$$

Assumed travelling wave solution (13) can be rewritten as follows:

$$\varepsilon(x,t) \propto \mathrm{e}^{\mathrm{i}[kx-\Omega(k)t]} \propto \mathrm{e}^{\mathrm{i}kx-\mathrm{i}\omega(k)t+\mu(k)t} \propto \boxed{\mathrm{e}^{\mu(k)t}} \cdot \mathrm{e}^{\mathrm{i}[kx-\omega(k)t]}. \tag{16}$$

It is easy to see that $\mu(k)$ acts as an exponential dissipation coefficient. The spectral components decay exponentially as $t \to \infty$ for $\mu(k) < 0$.

Dispersion and dissipation relations

$$\begin{cases} \omega(k) = \dfrac{\sqrt{6}}{12\delta S}\sqrt{\sqrt[3]{2}S^4 - 4S^2(1-3k^2\delta) + 2\sqrt[3]{4}(1-3k^2\delta)^2}, \\ \mu(k) = \dfrac{1}{12\delta S}\left[\sqrt[3]{4}S^2 - 4S + 2\sqrt[3]{2}(1-3k^2\delta)\right], \end{cases} \tag{17}$$

where

$$S = \sqrt[3]{2 - 9k^2\delta(1-3\delta) + 3k\delta\sqrt{3Q}}, \tag{18}$$

$$Q = 4k^4\delta - k^2(1 + 18\delta - 27\delta^2) + 4. \tag{19}$$

Three distinct regimes of solutions exist depending on the value of δ:

1. $0 < \delta \leqslant 1/9$: **High hereditary properties** accompanied by a band gap (BG) and a region with negative group velocity (NGV)
2. $1/9 < \delta < 1$: **Low hereditary properties** with continuous and smooth curves
3. $\delta = 1$: **Non-hereditary, dispersionless case**

The region with NGV exists for $0 < \delta \lesssim 0.134$ and a BG exists for $0 < \delta < 1/9$.

Dispersion and dissipation curves for $\delta = 0.05$

Case 1: **High hereditary properties**. Case corresponds best to wool felt because for wool felt $\gamma = 0.95$.

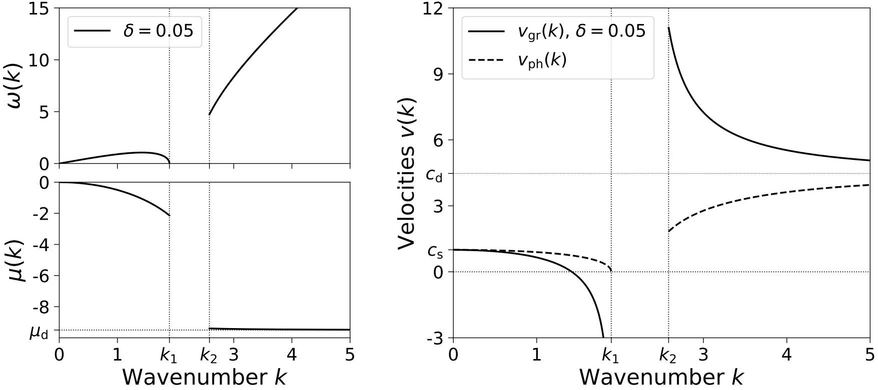


Figure: Dispersion and dissipation curves and corresponding phase and group velocity curves. This case features both normal ($v_{\mathrm{ph}}(k) > v_{\mathrm{gr}}(k)$) and anomalous ($v_{\mathrm{ph}}(k) < v_{\mathrm{gr}}(k)$) dispersion types.

Dispersion and dissipation curves for $\delta = 1/9 \approx 0.11$

Case at the boundary that separates Case 1 from Case 2.

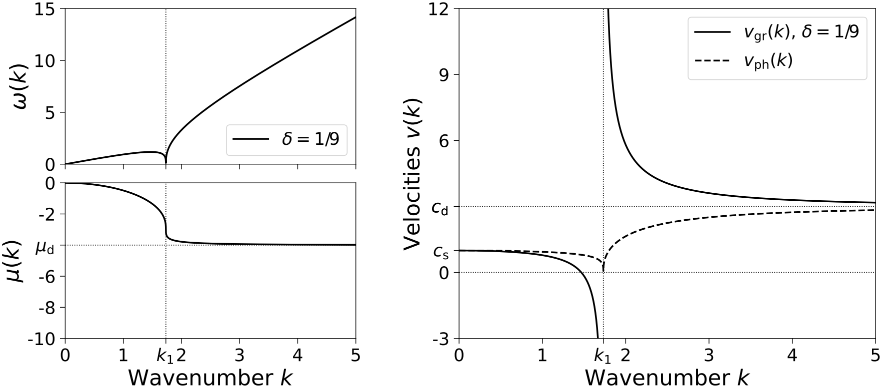


Figure: Dispersion and dissipation curves and corresponding phase and group velocity curves. This case features both normal ($v_{\rm ph}(k) > v_{\rm gr}(k)$) and anomalous ($v_{\rm ph}(k) < v_{\rm gr}(k)$) dispersion types.

Dispersion and dissipation curves for $\delta = 0.2$

Case 2: **Low hereditary properties**

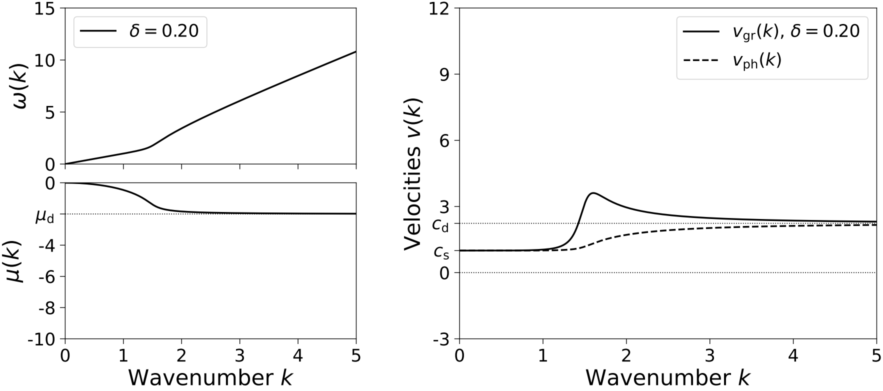


Figure: Dispersion and dissipation curves and corresponding phase and group velocity curves.

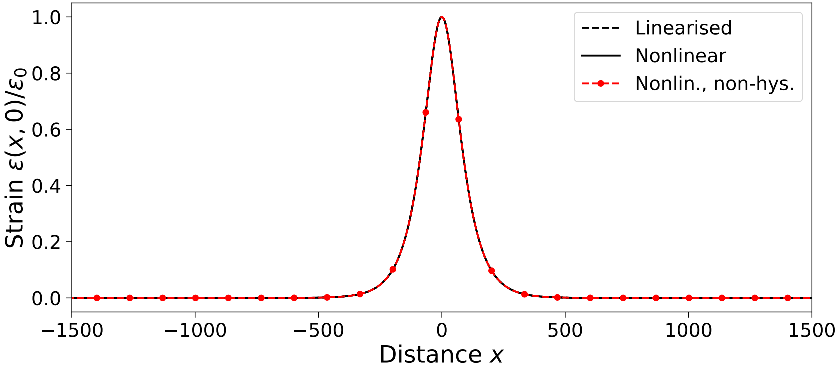
Comparison of models: *Low frequency* pulse

Initial value problem (IVP) applied to three models:

$$\varepsilon_{tt} = \varepsilon_{xx} + \varepsilon_{xxt} - \delta\varepsilon_{ttt}, \qquad \varepsilon_{tt} = (\varepsilon^p)_{xx} + (\varepsilon^p)_{xxt} - \delta\varepsilon_{ttt}, \qquad \varepsilon_{tt} = \delta^2(\varepsilon^p)_{xx},$$

$$\varepsilon(x,0) = \varepsilon_0 \operatorname{sech}(\beta x), \tag{20}$$

where $\delta = 0.05$, $p = 1.1$, $\beta = 0.015$ is the pulse width parameter, the corresponding spectral component $k \approx 2$. Resulting phase speed is δ.



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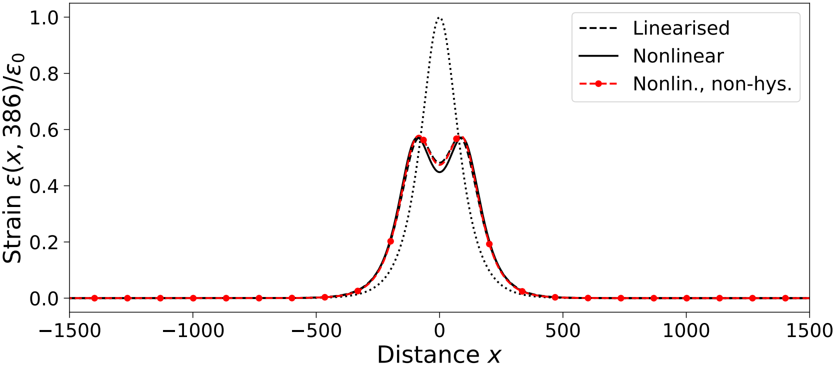

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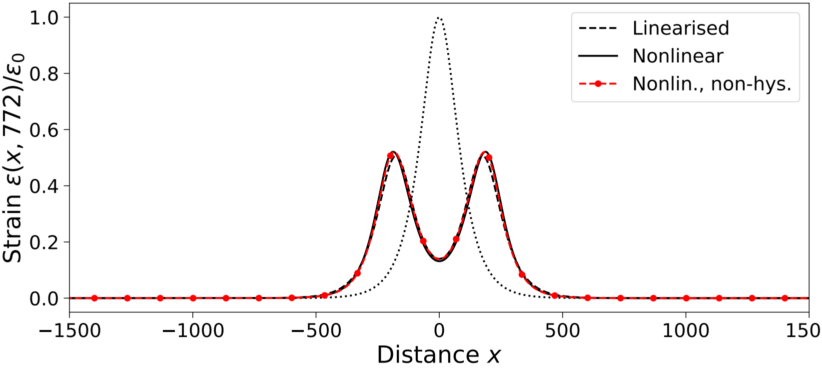

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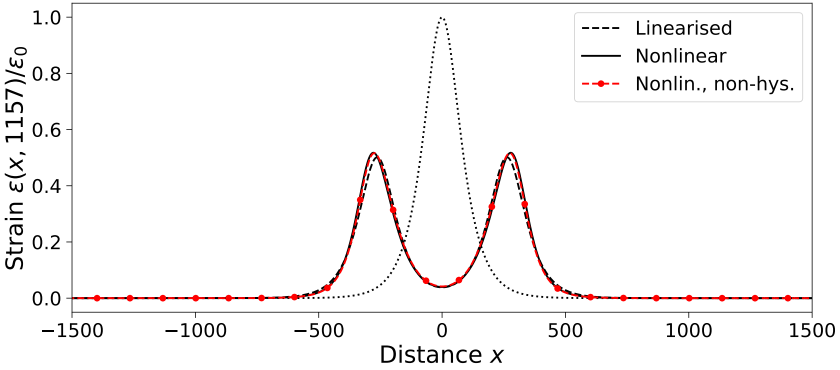

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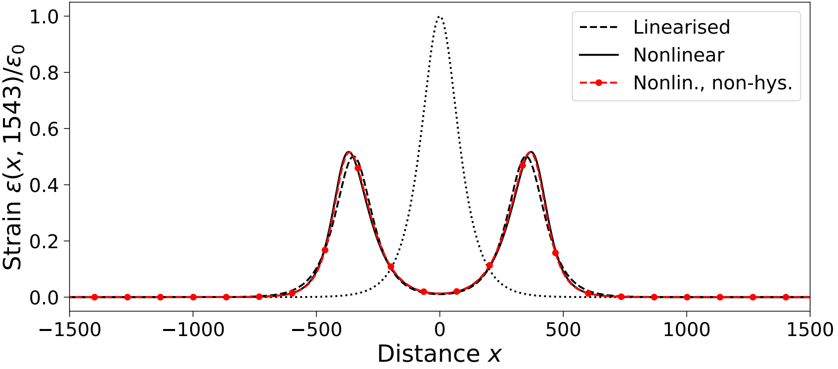

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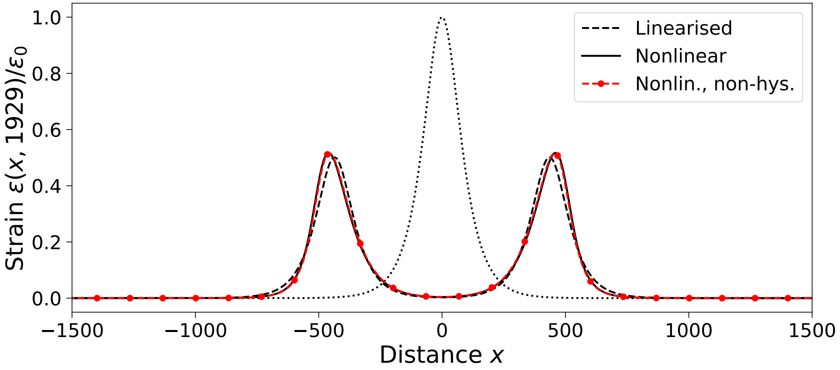

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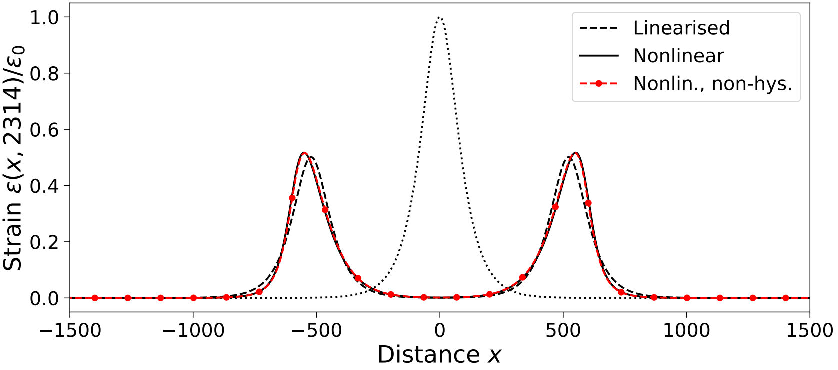

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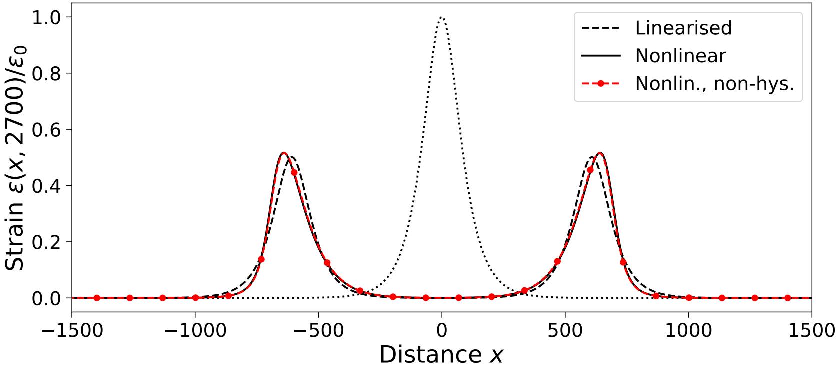

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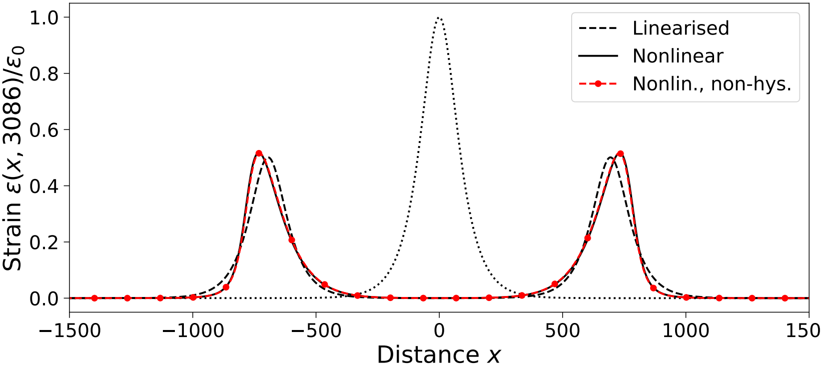

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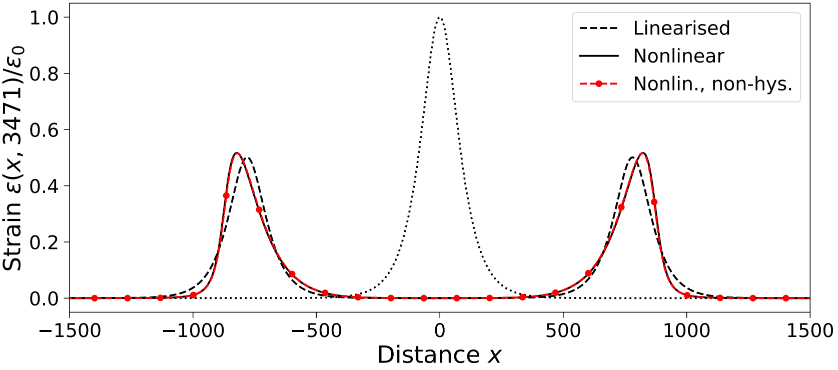

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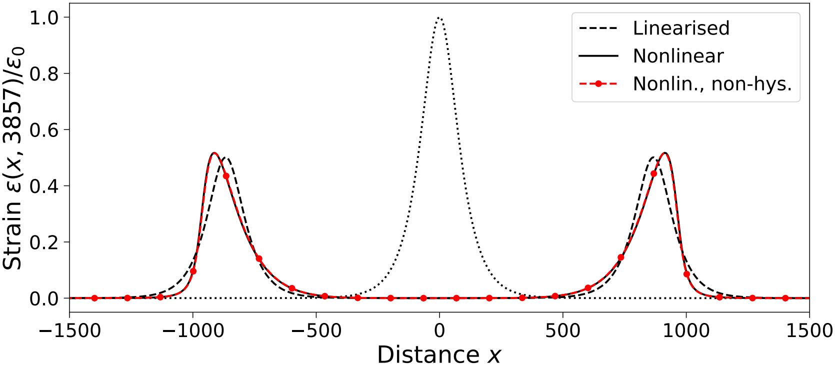

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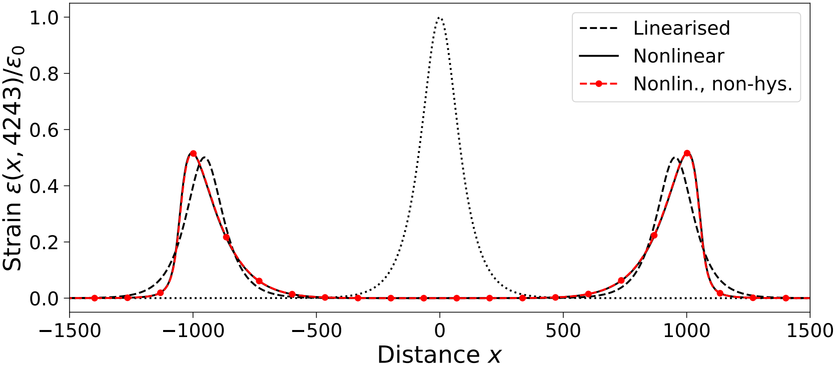

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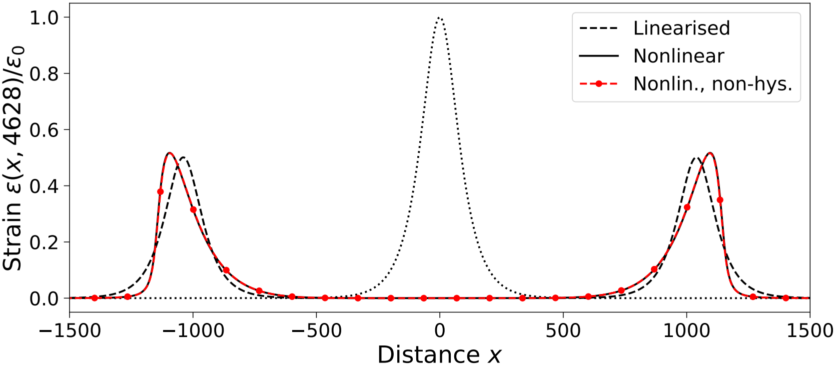

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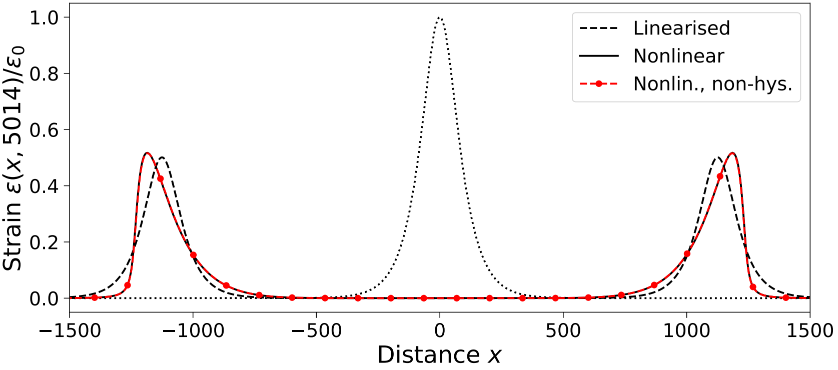

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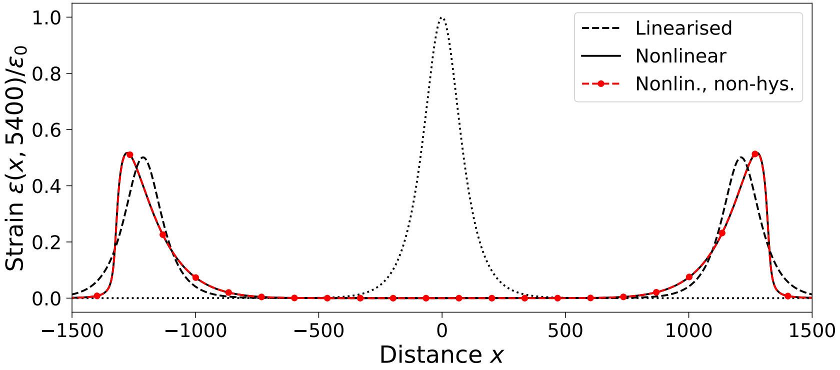

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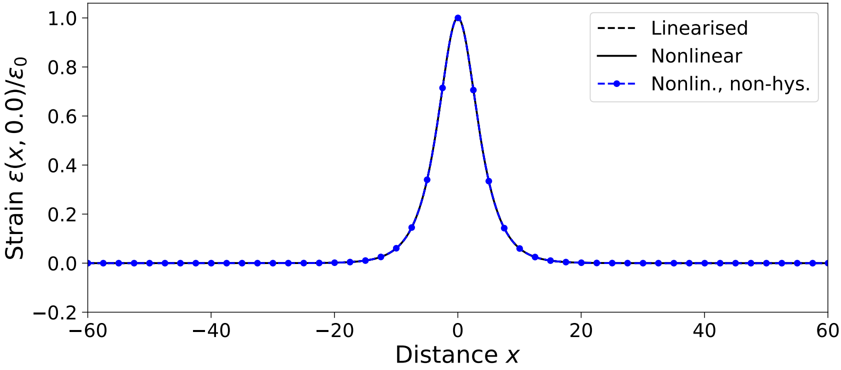
$$\varepsilon(x,0) = \varepsilon_0 \,\mathrm{sech}(\beta x),$$

where $\delta = 0.05$, $p = 1.1$, $\beta = 0.35$, the corresponding spectral component $k \approx 60$. Resulting phase speed is δ.



Comparison of models

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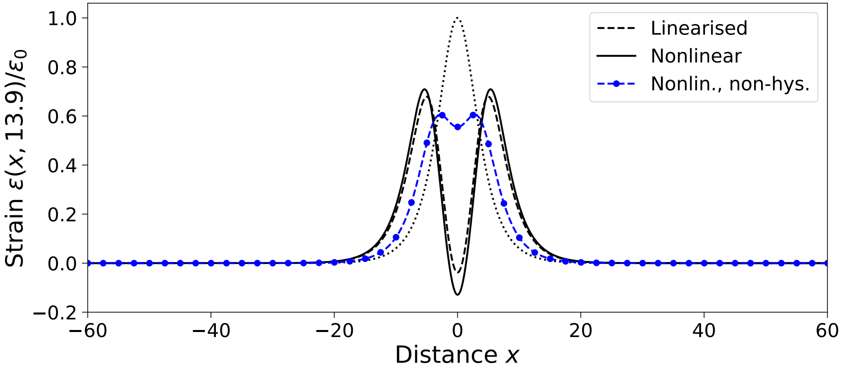

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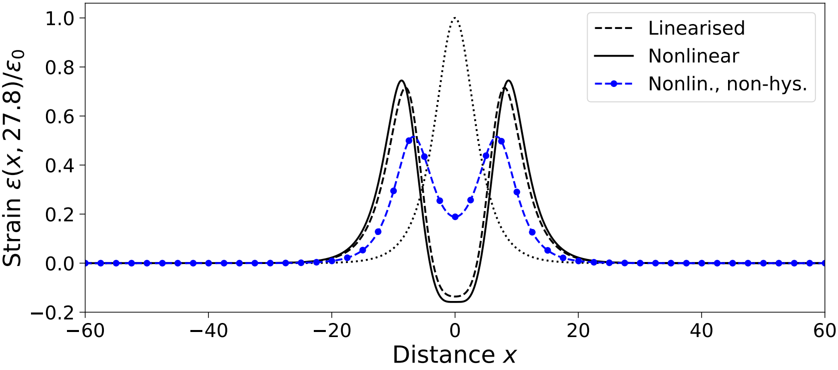

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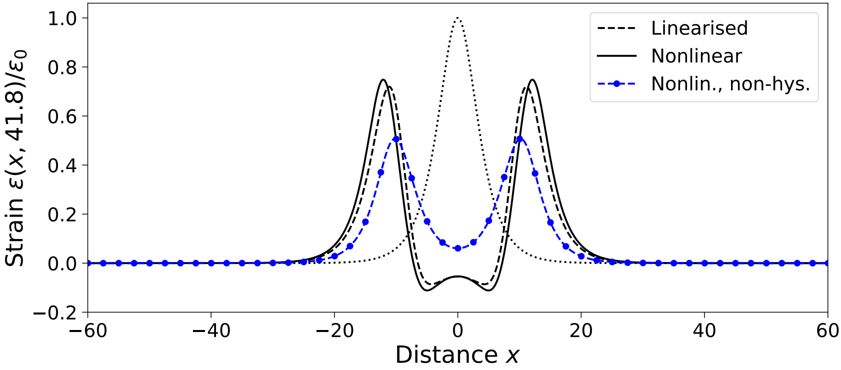

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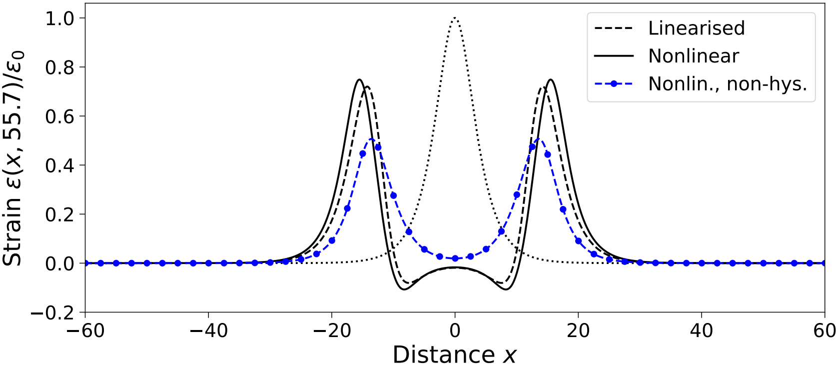

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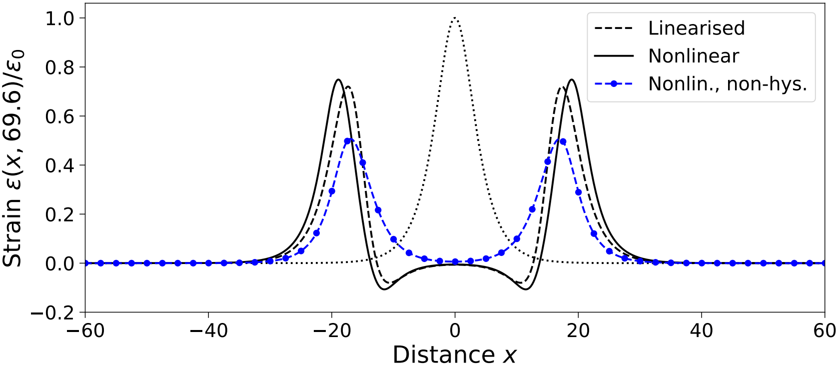

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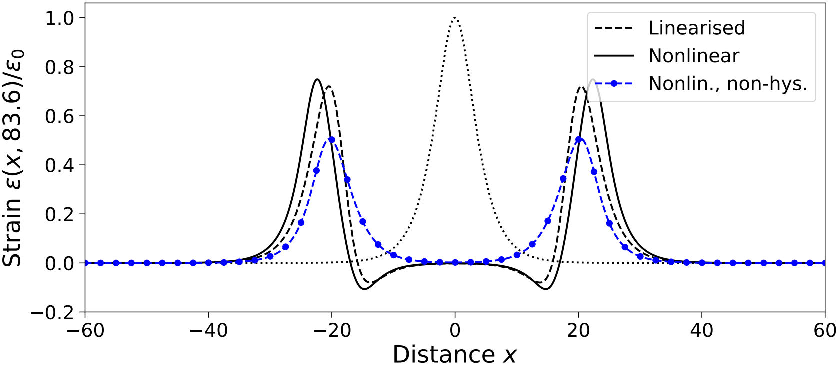

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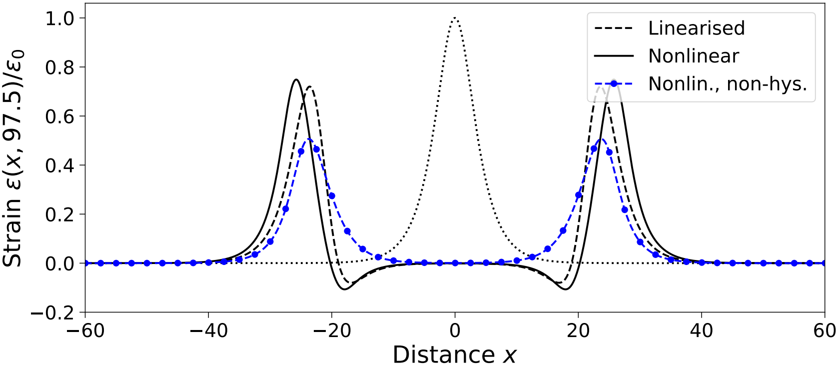

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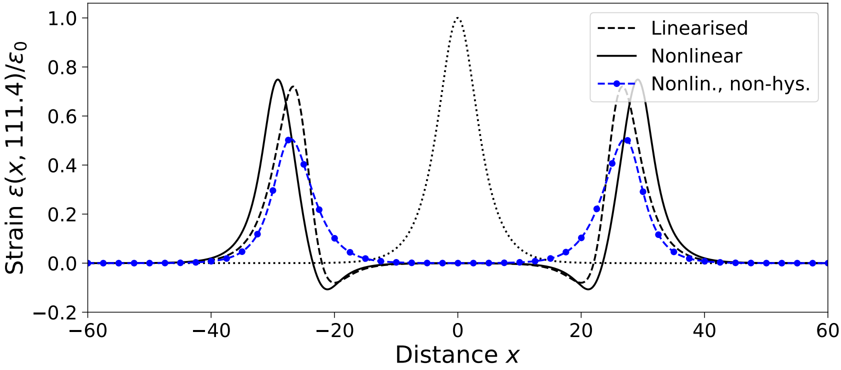

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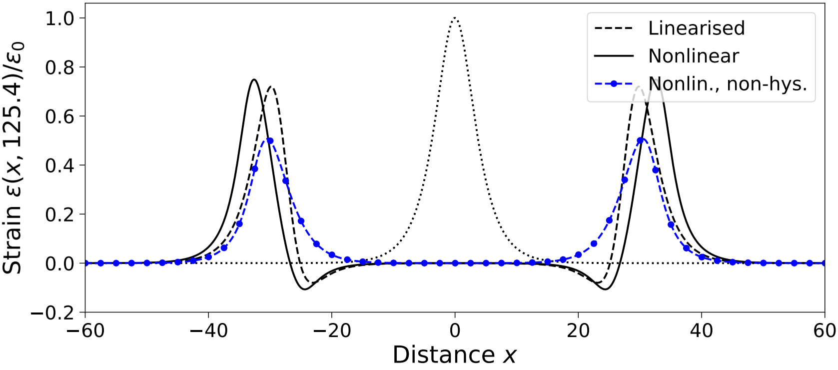

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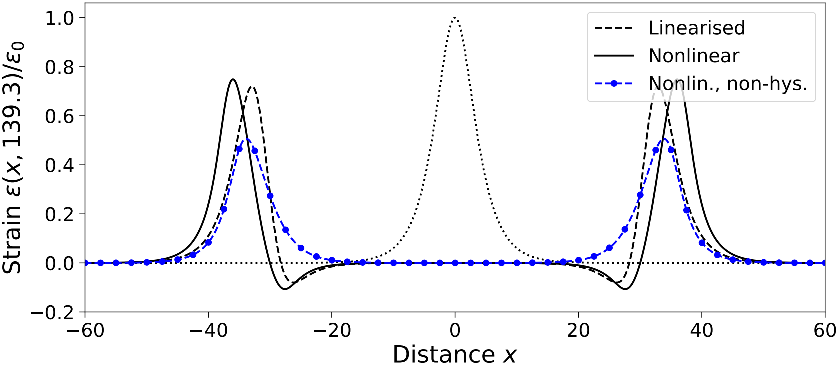

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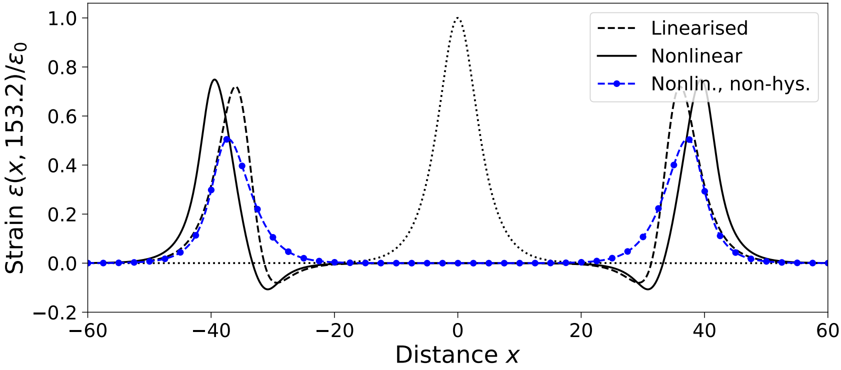

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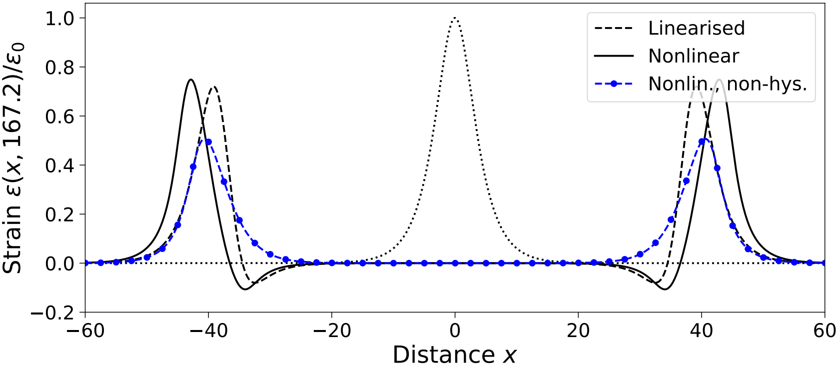

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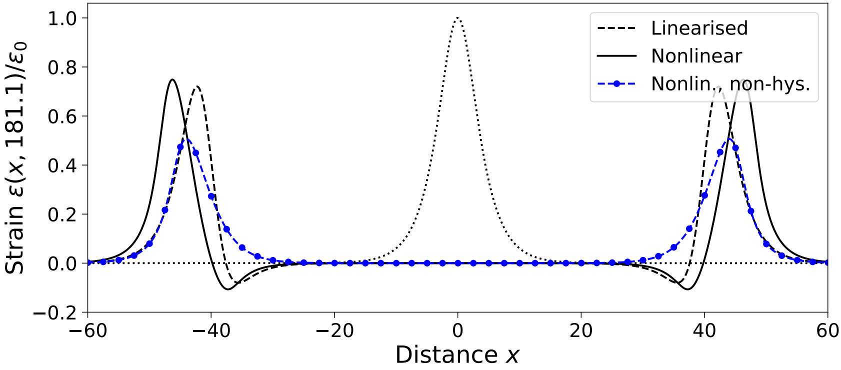

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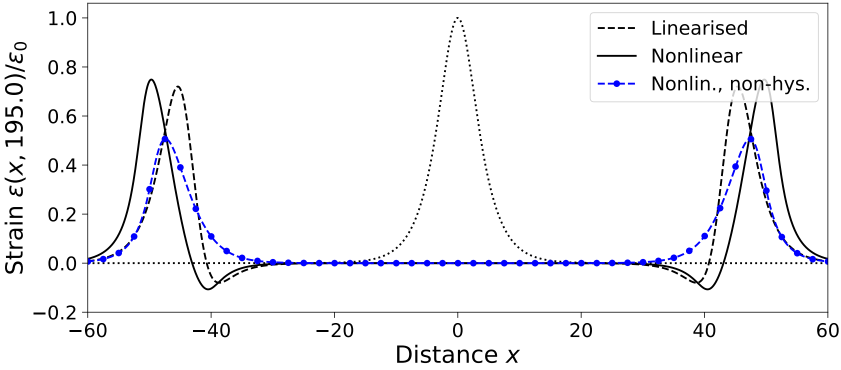

Conclusions

- An equation for modelling strain wave propagation through felt and similar materials was presented.
- Dispersion and dissipation analysis was performed. Surprisingly, dispersion curves featured a region with NGV and a BG.
- Three distinct solution behaviour regimes were identified.
- A *naive interpretation* of dispersion relation may lead to wrong conclusions about the underlying model and physics.
- Despite being predicted by the dispersion analysis the BG and NGV may not influence the wave evolution. Wave lengths related to the NGV and BG spectral components were *too big* relatively speaking. If the material loading and unloading time-scale is *much too great* in comparison to material relaxation time τ_0, then any imaginable effects will be negligibly small. Similar masking effect also influenced the dissipation.

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ETAG

On pulse propagation in poro-visco-elastic felt-like material

Dmitri Kartofelev, PhD   Maria M. Vuin, MSc

Tallinn University of Technology,
School of Science, Department of Cybernetics, Laboratory of Solid Mechanics,
Tallinn, Estonia

Friday, Nov. 10, 2023

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