Sorrento SAPEM'23 常熟

3D Auxetic Lattice Metamaterials From Distorted Kelvin Cells

Huina Mao[1,2,3,*], Romain Rumpler[1,2], **Peter Göransson**[1,2], Gunnar Tibert[1,3]

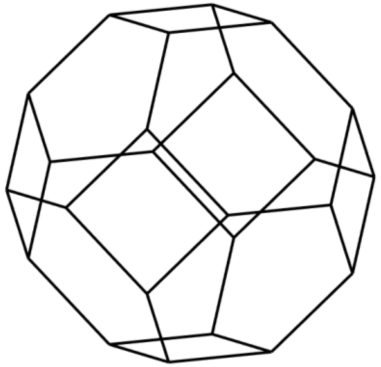
[1]KTH Royal Institute of Technology, Stockholm, Sweden

[2]MWL - The Marcus Wallenberg Laboratory for Sound and Vibration

[3]KTH Space Center

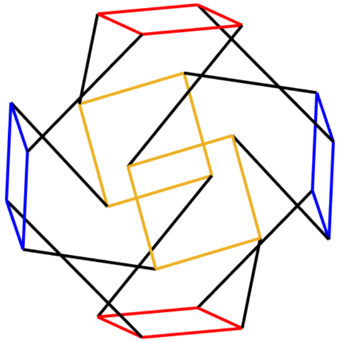
huina@kth.se

Isometric Kevin Cell

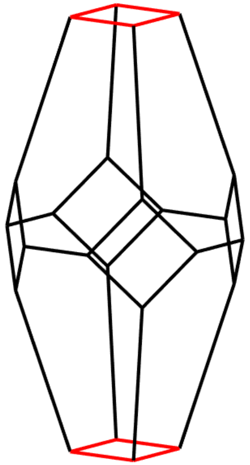


Geometric Distortions

Twist



Stretch

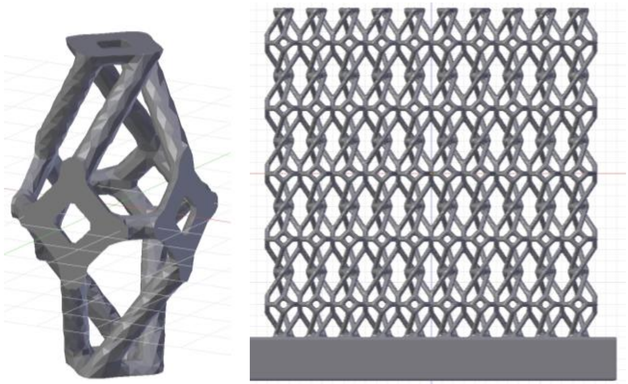


Inverse Characterisation

$$\sigma = \mathbf{H}\epsilon$$

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ & & & H_{44} & H_{45} & H_{46} \\ & sym & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{bmatrix}$$

Elastic Material Properties



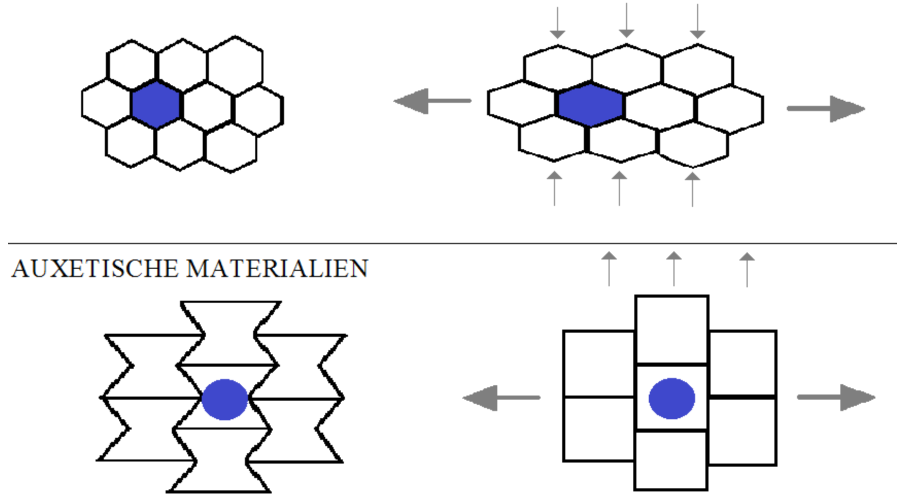
Additive Manufacturing

Experimental Tests and Simulations

Auxetic

Negative Poisson’s ratio

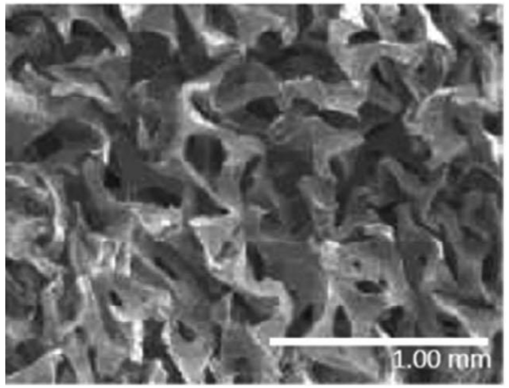
Non-auxetic **Stretch - thinner**



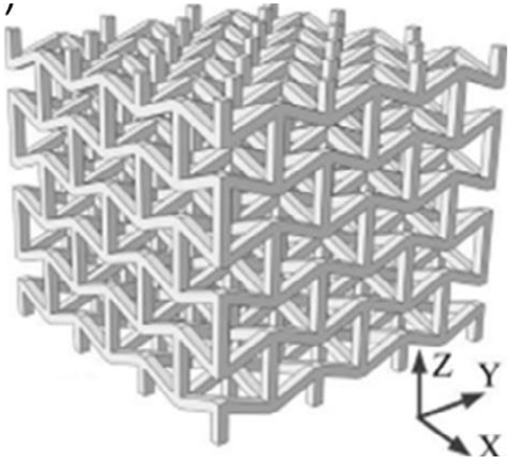
Auxetic Material **Stretch - thicker**

Deformation schematics of non-auxetic and auxetic structures.

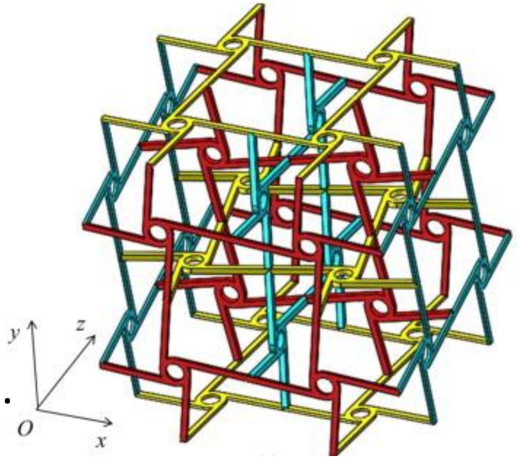
3D Auxetic structures



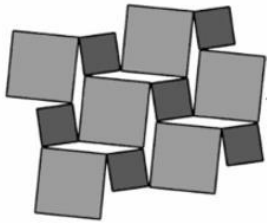
Auxetic foams



Re-entrant structures



Chiral structures



Others (rotating, foldable structures, etc.)

Motivation

Current design of 3D auxetic structures

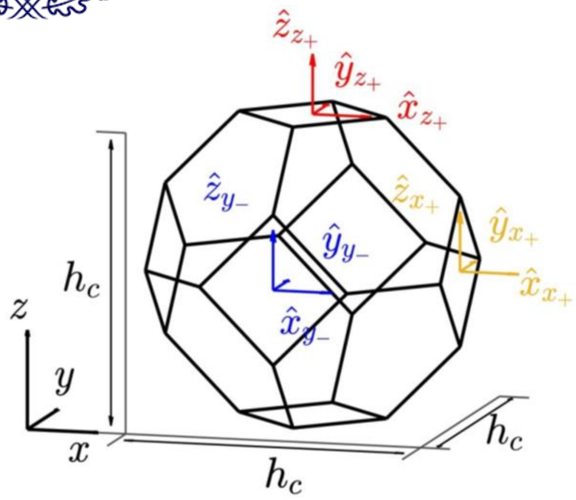
- Negative Poisson's effect is limited to one/two directions
- Pure auxetic effect that decoupled with shearing behaviors.

Target design of 3D auxetic structures

- Programmable negative Poisson's effect in all three directions.
- Controllable auxetic effect that could be coupling or decoupling with shearing behaviors – Anisotropic auxetic structures.

Part I: Design

New 3D Lattice Structures



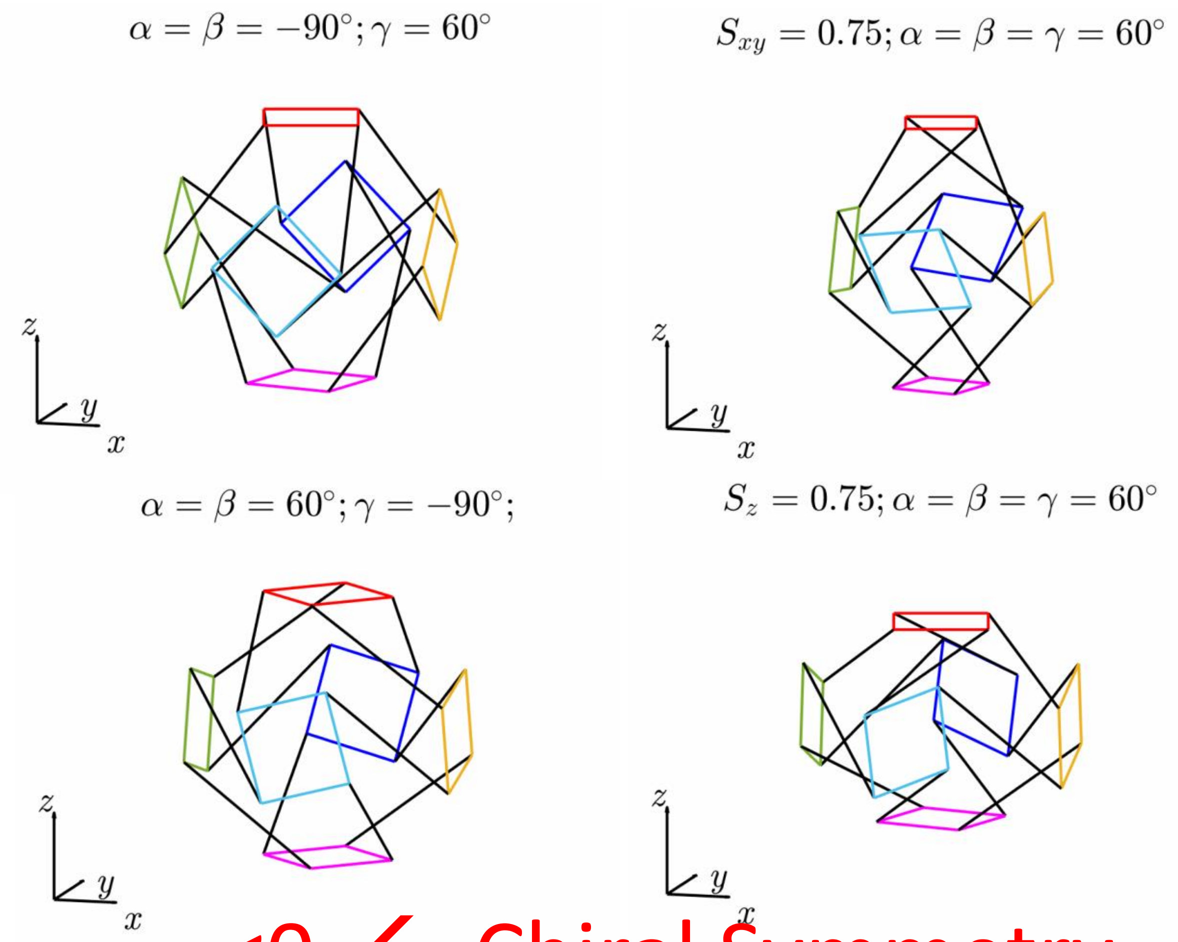
Kelvin Cell

Quadrilateral Faces
x+, x-, y+, y-

Quadrilateral Faces
z+, z-

- **Around z-axis by $\pm 90^{\circ}$**
- **Around axis $[1, 1, 0]$ by $\pm 180^{\circ}$**

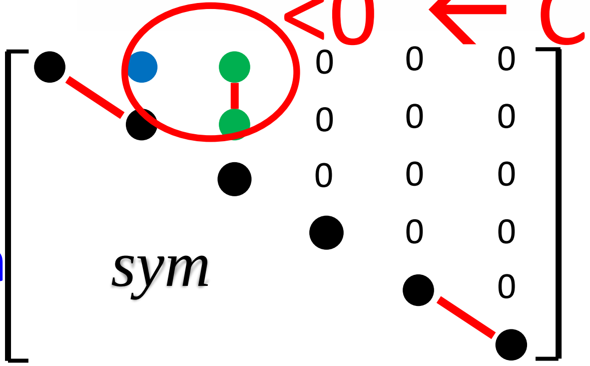
Twist | **Stretch**



$$
\mathbf{H}=\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ & & & H_{44} & H_{45} & H_{46} \\ & sym & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{bmatrix}
$$

2 Rotational Symmetries

No Mirror Plan



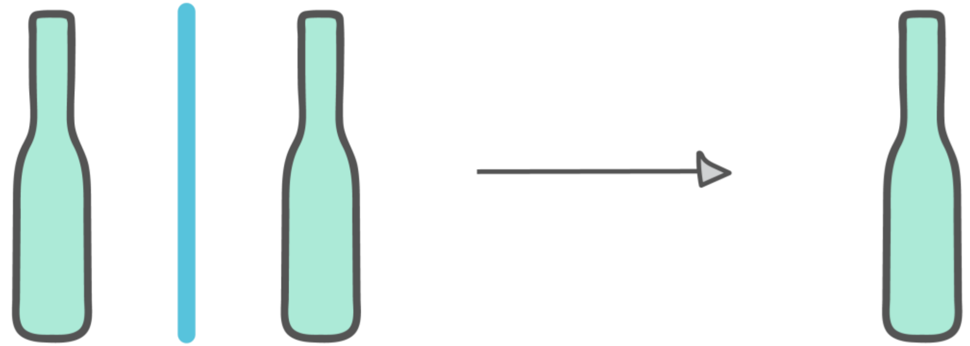
← **Chiral Symmetry**

Note:
6 independent elastic constants
$G_{xy} \neq E_x/2(1+\nu_{xy})$
Not transverse isotropy

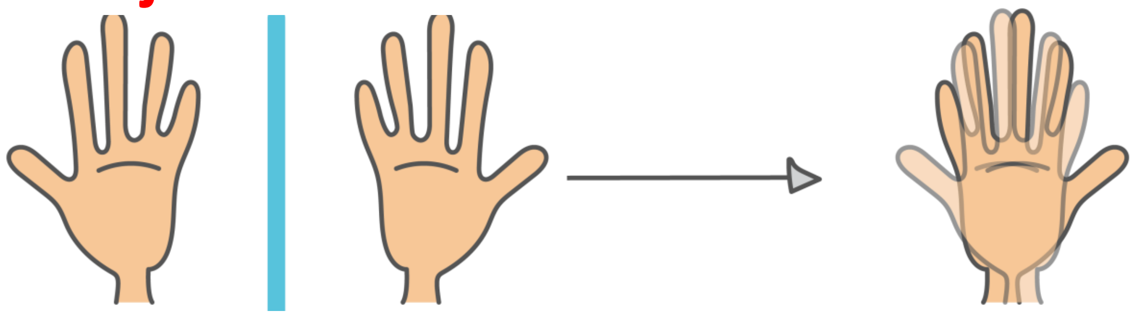
Chiral symmetry

Achiral objects

Can be superimposed



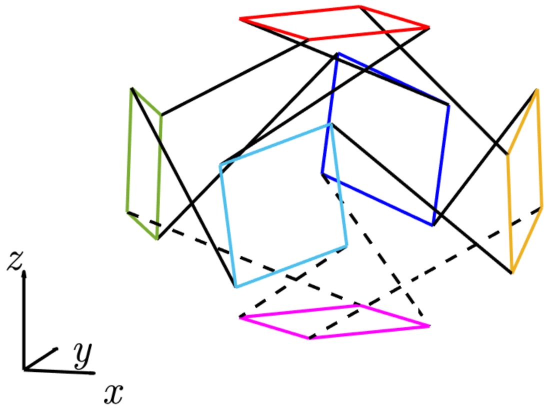
Chiral objects



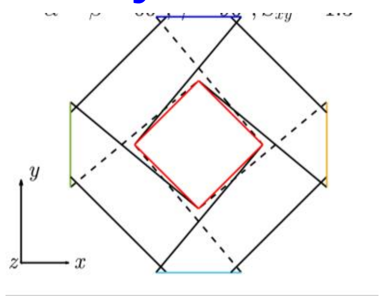
Left hand Mirror Right hand

Can not be superimposed

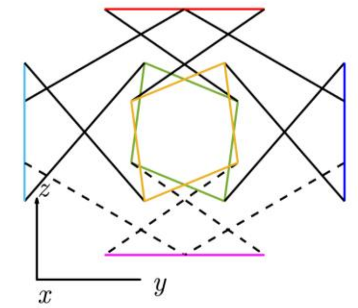
$\alpha = \beta = 60°; \gamma = 90°; S_{xy} = 1.3$



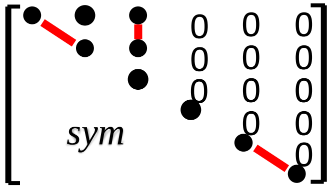
x-y view



y-z view



- No Mirror Plane
- 2 Rotational Symmetry
  - **z-axis by $\pm 90^{\circ}$**
  - **Axis $[1,1,0]$ by $\pm 180^{\circ}$**



(6 Independent Parameters, 0 mirror face)

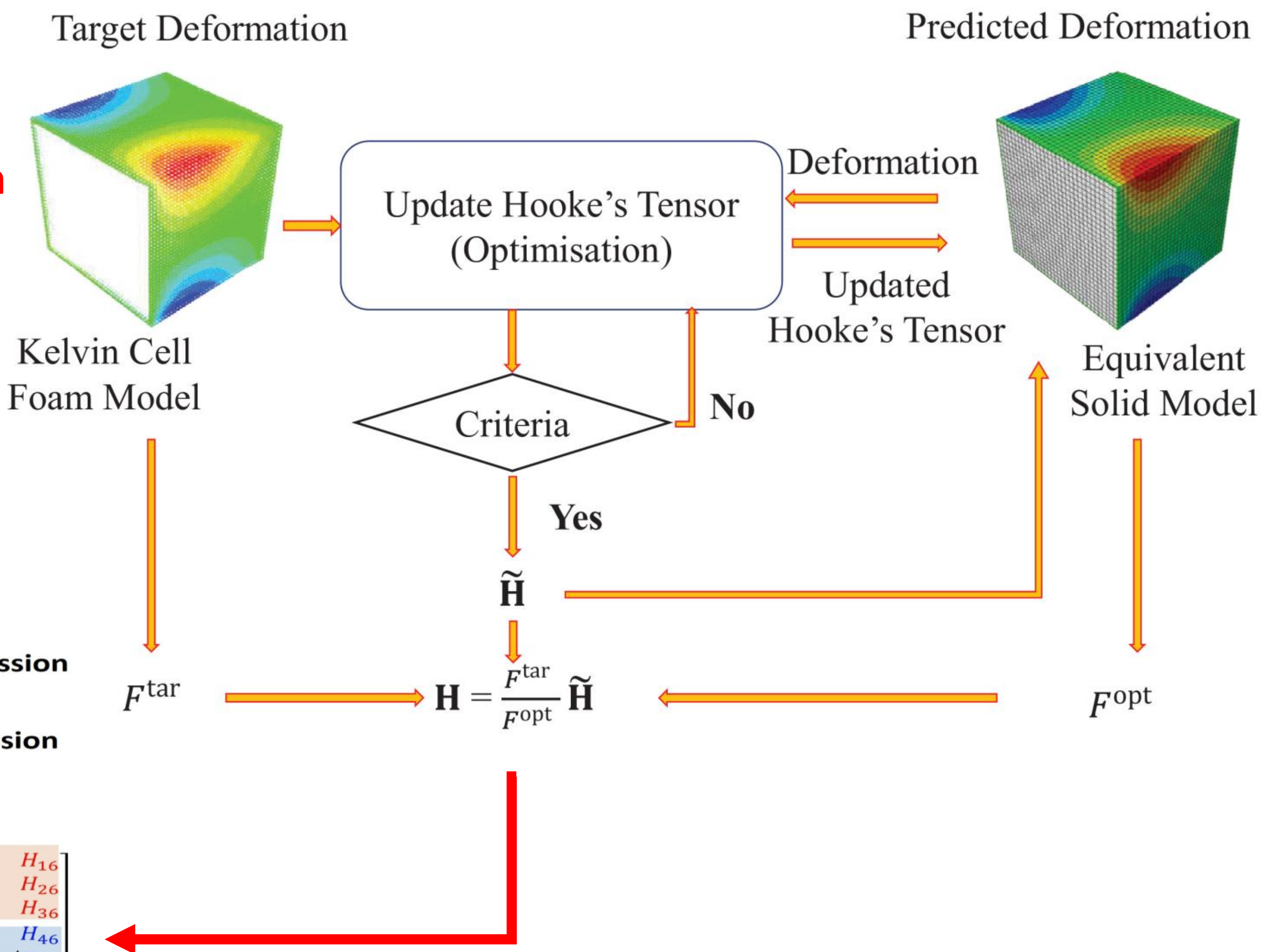
Non-central forces

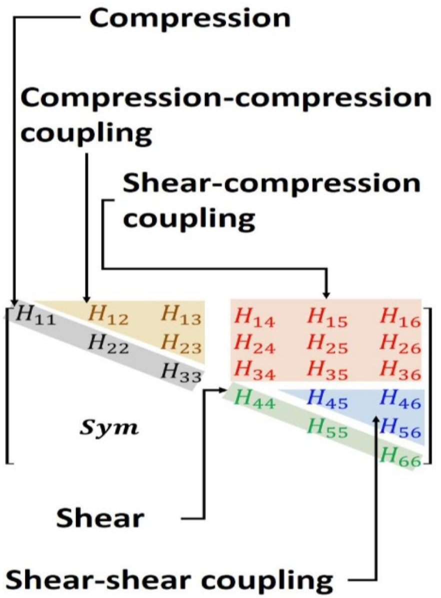
Negative Poisson's ratios

Part II: Characterize Hooke’s Matrix

Inverse Characterization Method

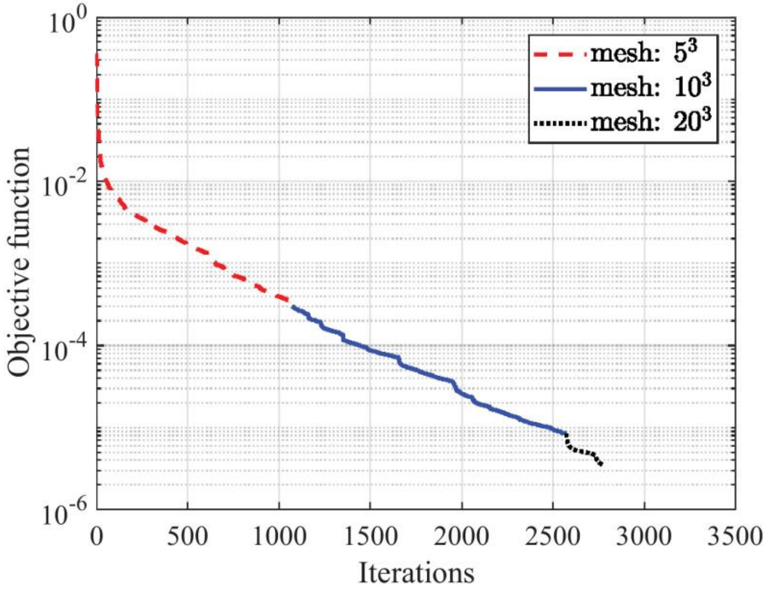
Distorted Kelvin cell foam





Cost objective function

$$\Pi_{\text{total}}(\mathbf{X}) = \sum_{c=1}^{N_{lc}} \sum_{f=1}^{4} \frac{\left\| {}_f^c\mathbf{u} - {}_f^c\mathbf{U} \right\|_2^2}{\left\| {}_f^c\mathbf{U} \right\|_2^2}$$



Twist Z faces:

$\alpha = \beta = 0;\ \gamma = \phi$

6 independent elastic constants

Twist Z + Y faces:

$\alpha = 0;\ \beta = \gamma = \phi$

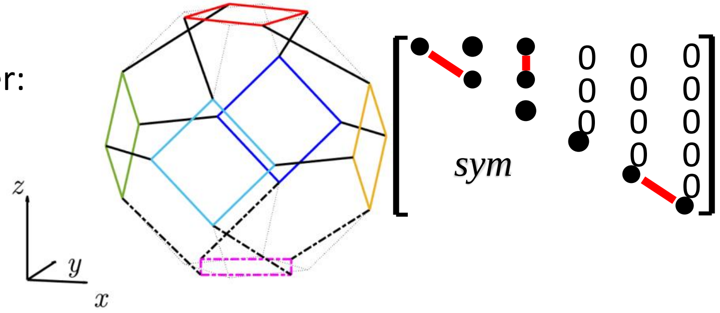
6 independent elastic constants

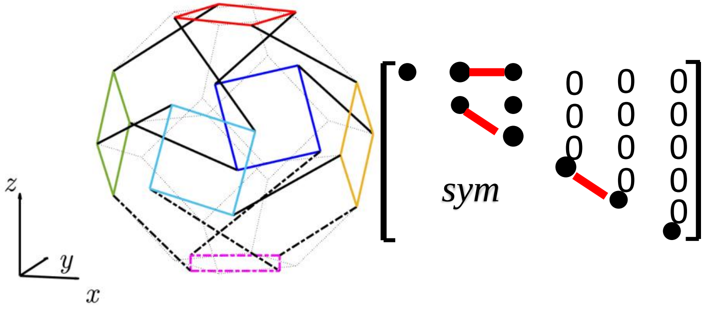
Twist Z + Y + X faces:

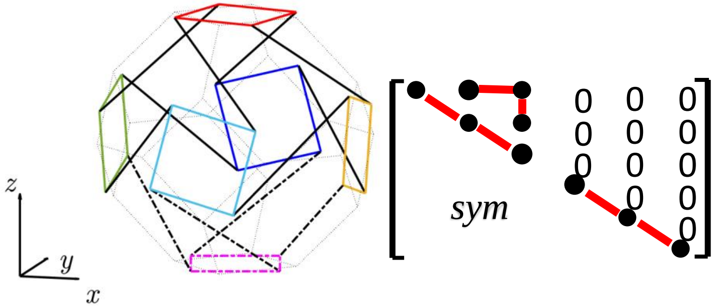
$\alpha = \beta = \gamma = \phi$

3 independent elastic constants

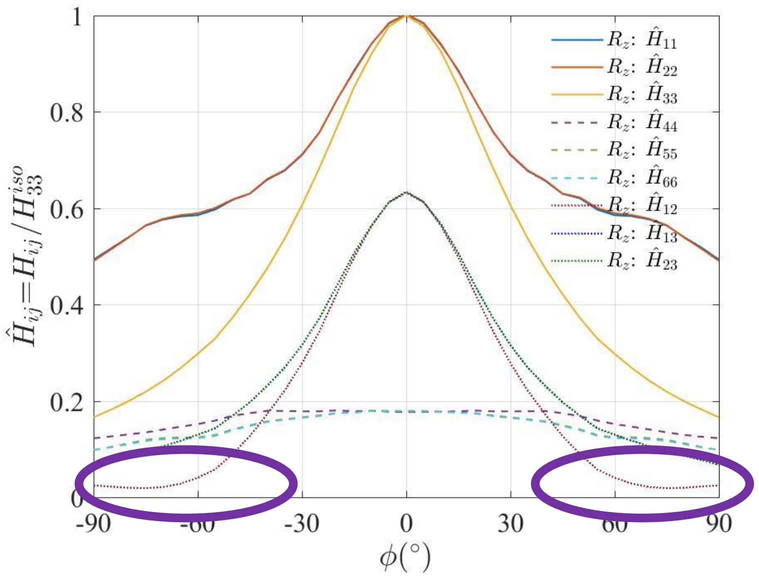
Cell size:
$h_c = 3$ mm
Struss diameter:
$d_s = 0.5$ mm

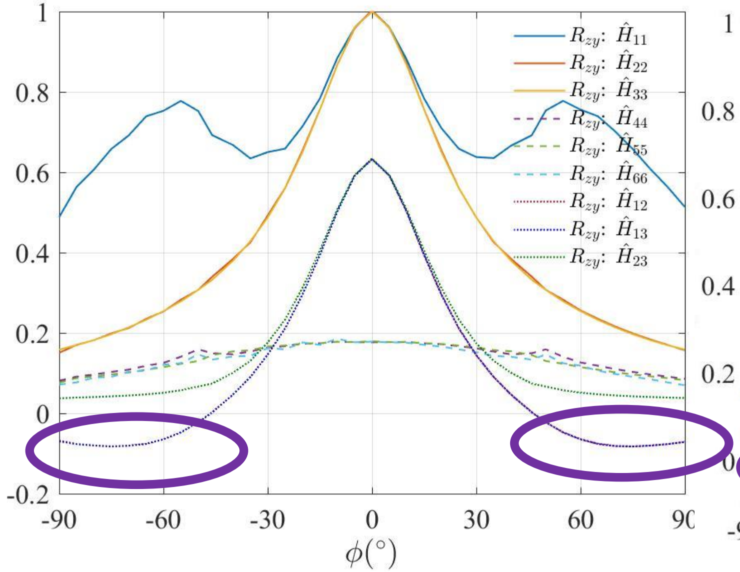


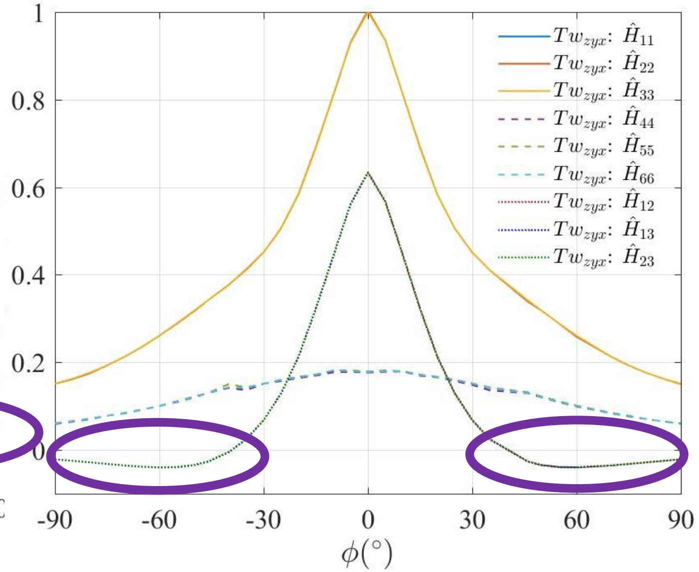




Inverse Characterized Elastic Constants: Normalized to H_{33} of isometric KC



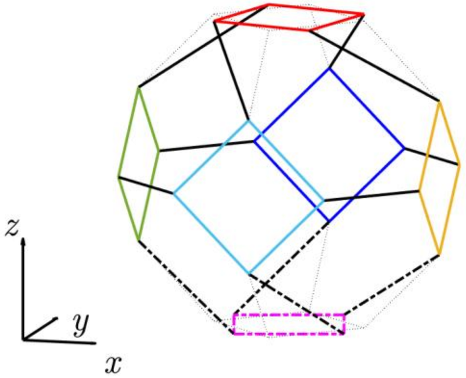


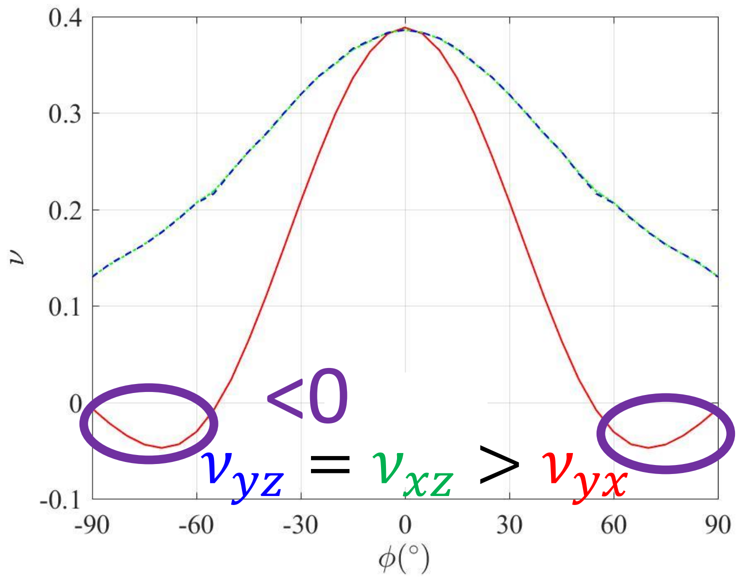


Equivalent Poisson's ratios

Twist Z:

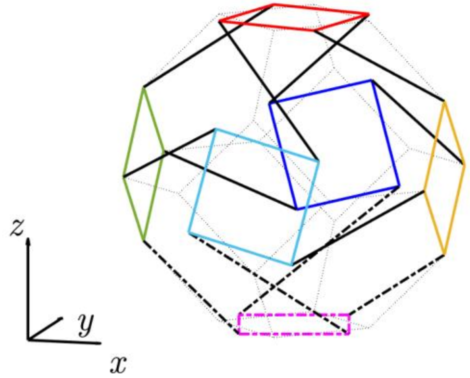
$\alpha = \beta = 0; \gamma = \phi$

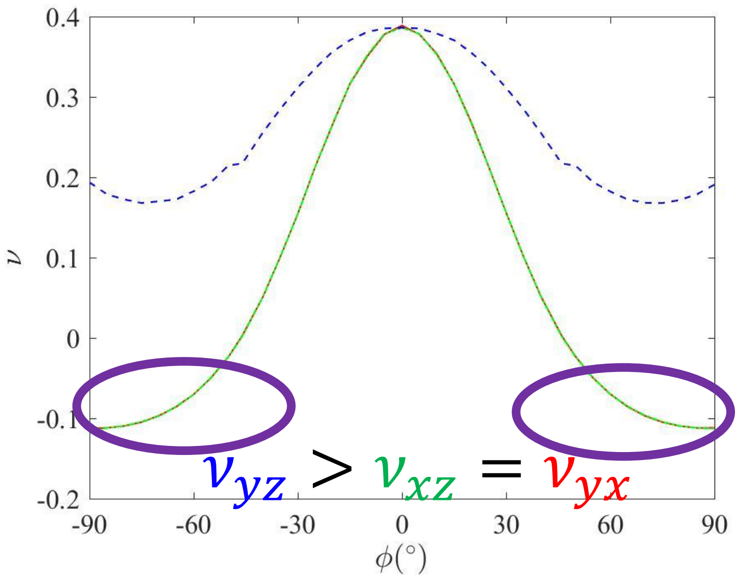




Twist Z+Y:

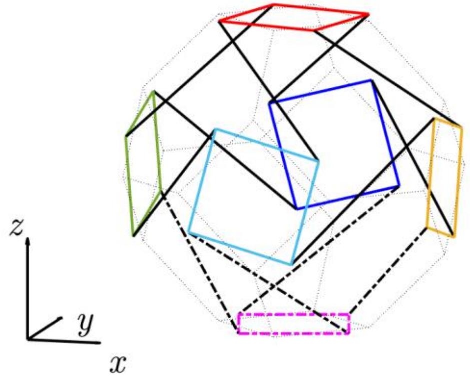
$\alpha = 0; \beta = \gamma = \phi$

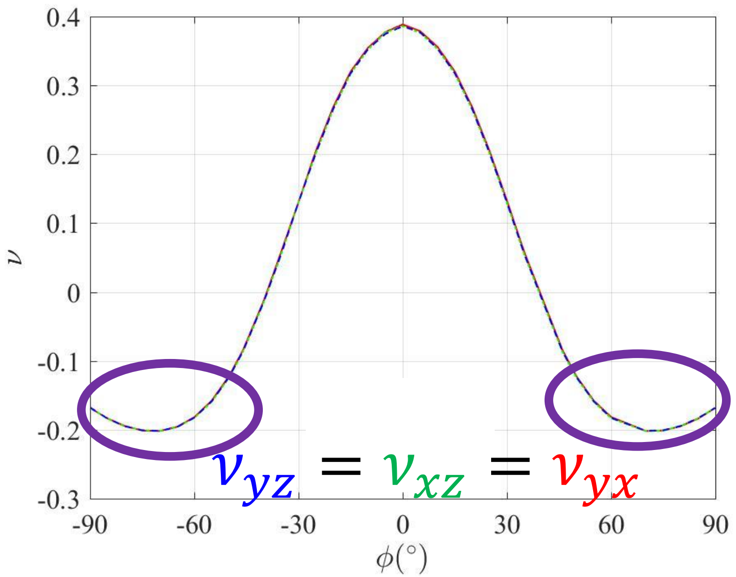




Twist Z+Y+X:

$\alpha = \beta = \gamma = \phi$

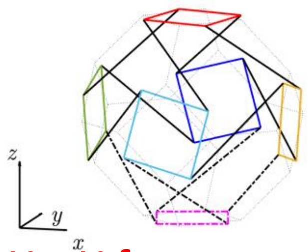




$$H = f\left(\frac{d_s}{h_c}, E_{solid}, others\right)$$

Diameter of the beams of the cell (d_s)

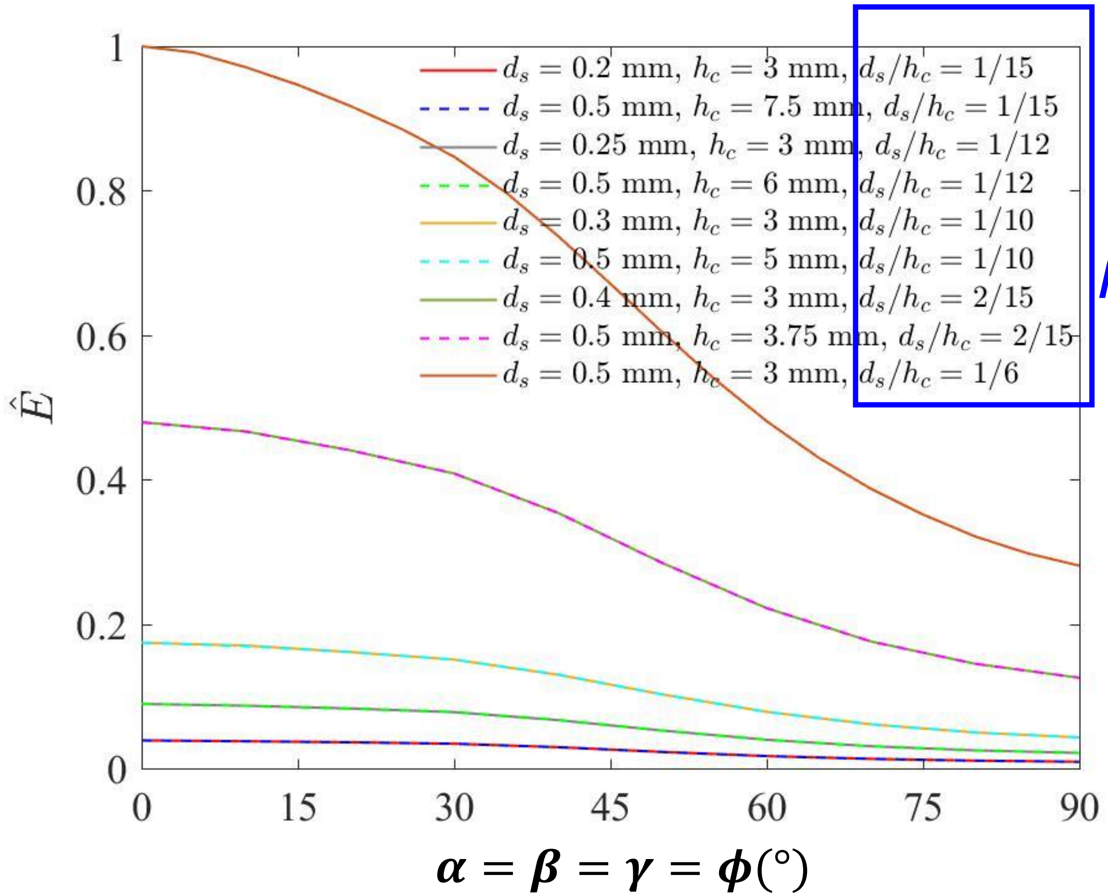
Size of the cell (h_c)



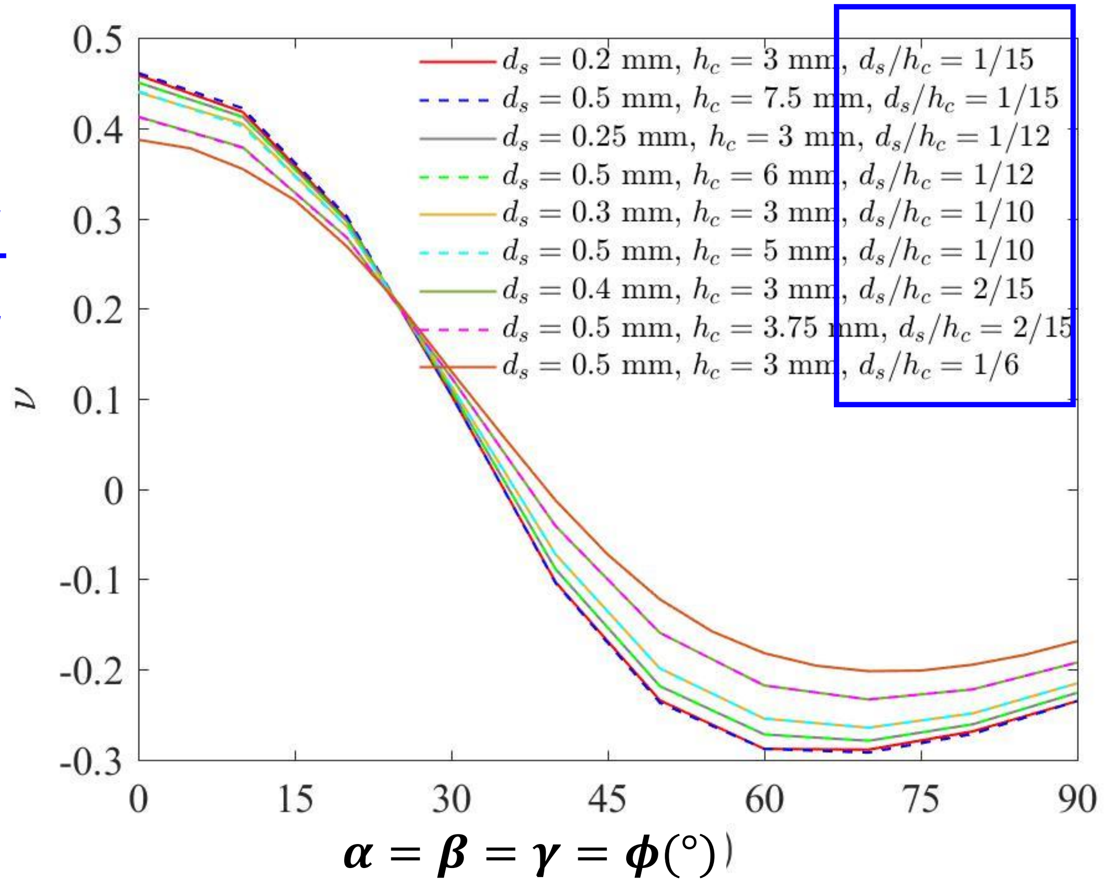
Twist Z + Y + X faces: $\alpha = \beta = \gamma = \phi$

Normalized Young's modulus: $\widehat{E} = E/E_{solid}$

Poisson's ratio: $\nu = \nu_{yz} = \nu_{xz} = \nu_{yx}$



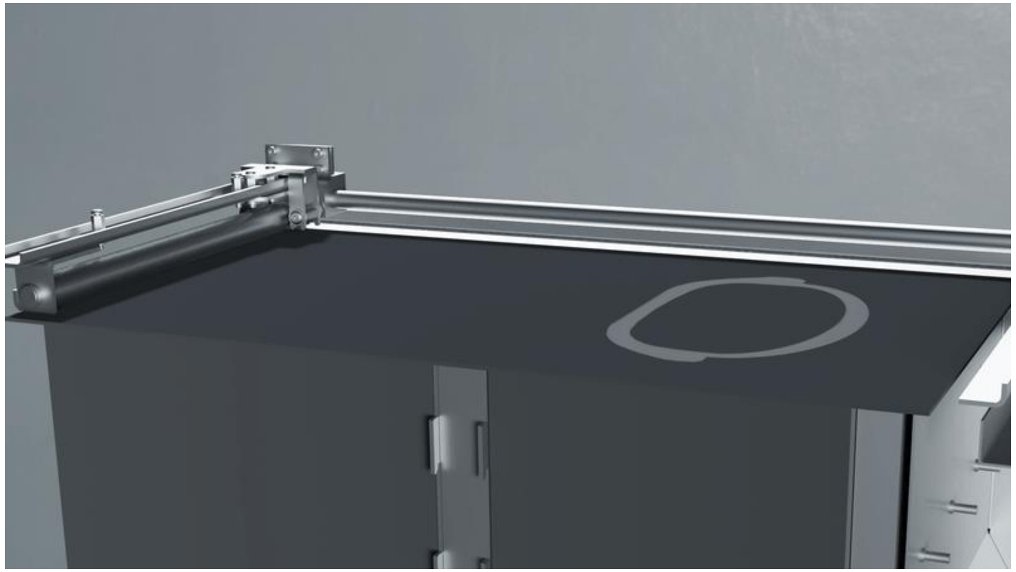
$$\rho = \frac{d_s}{h_c}$$



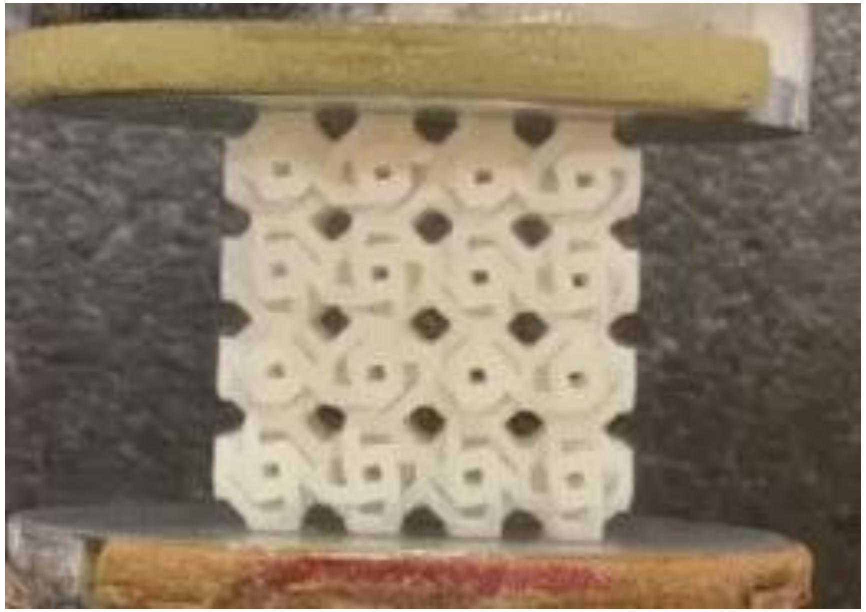
Part III: Additive Manufacturing & Tests

SLS - selective laser sintering

- SLS - a laser selectively sinters the particles of a polymer powder layer by layer.
  - ❑ No need for support materials: complex geometry.
  - ❑ Limited to the cell size and accuracy of the specimens.



- TPU Materials - Thermoplastic Urethane
  - ❑ High Elongation ~500%
  - ❑ Relatively lower compression modulus: Ex = 15 MPa, Ez = 20 Mpa, Poisson's ratio = 0.45
  - ❑ Selected to demonstrate large deformation shapes
  - ❑ Cheap for testing



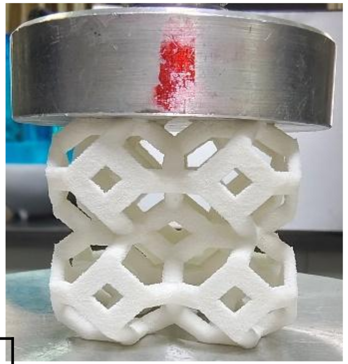
h_c = 10 mm
d_s = 1.5 mm

Compression – Z: Print direction (strain =50%)

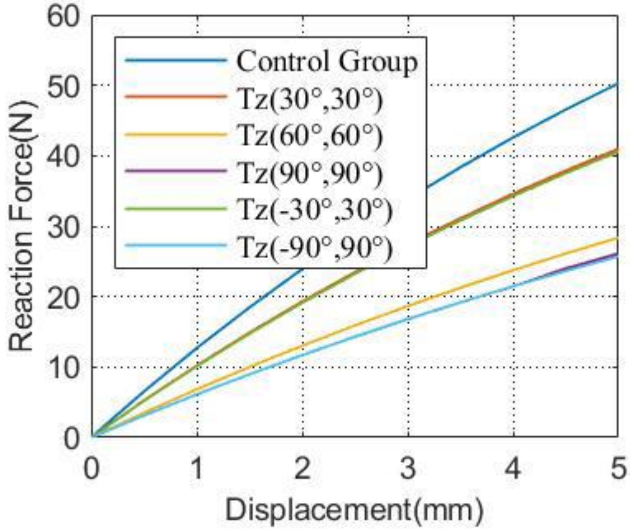
Simulations

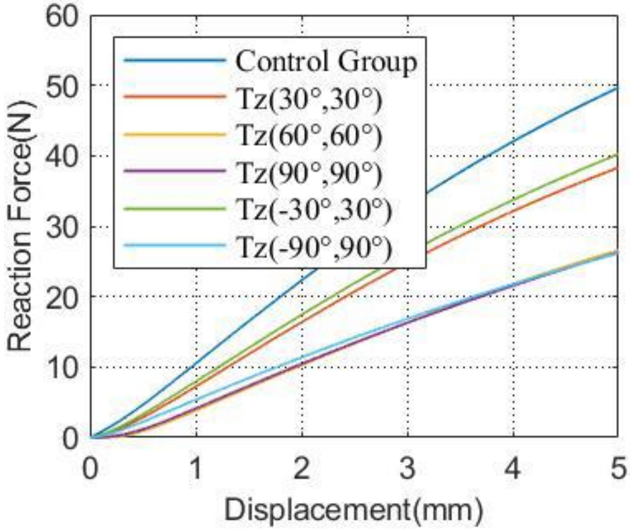
Experiments

Relative error (%)

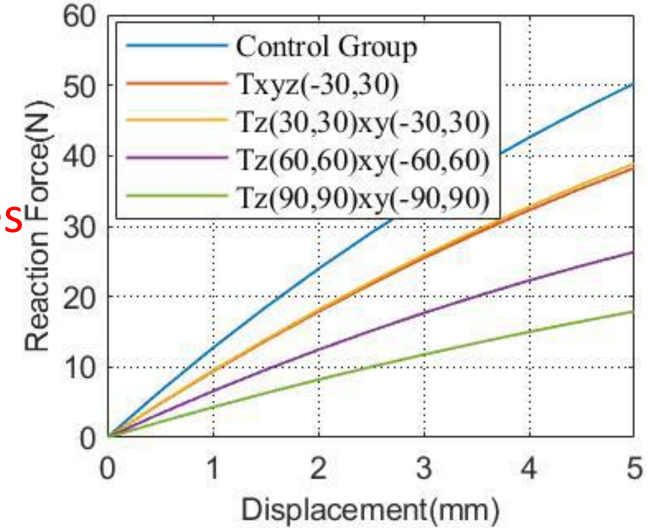


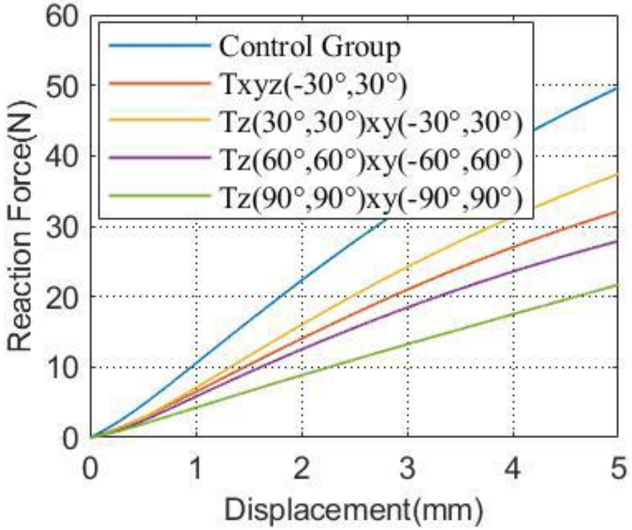
Twist Z faces





Twist Z, Y, X faces





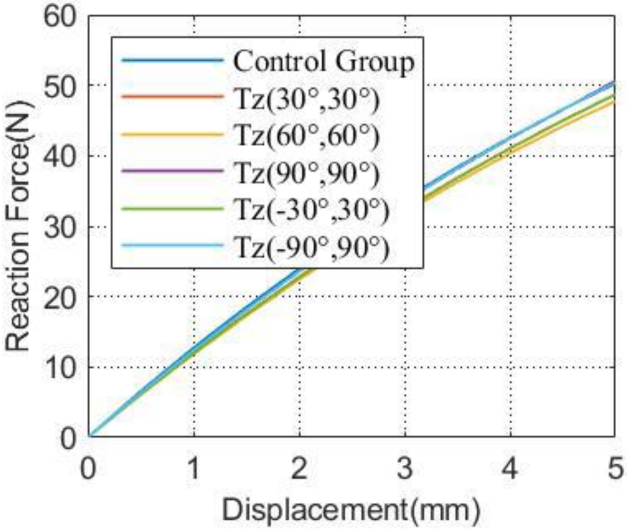
KC	1.06
$T_z(30°,30°)$	6.59
$T_z(60°,60°)$	6.17
$T_z(90°,90°)$	0.57
$T_z(-30°,30°)$	0.99
$T_z(-90°,90°)$	1.95
$T_{xyz}(-30°,30°)$	16.08
$T_z(30°,30°)$ $T_{xy}(-30°,30°)$	3.86
$T_z(60°,60°)$ $T_{xy}(-60°,60°)$	5.88
$T_z(90°,90°)$ $T_{xy}(-90°,90°)$	21.23

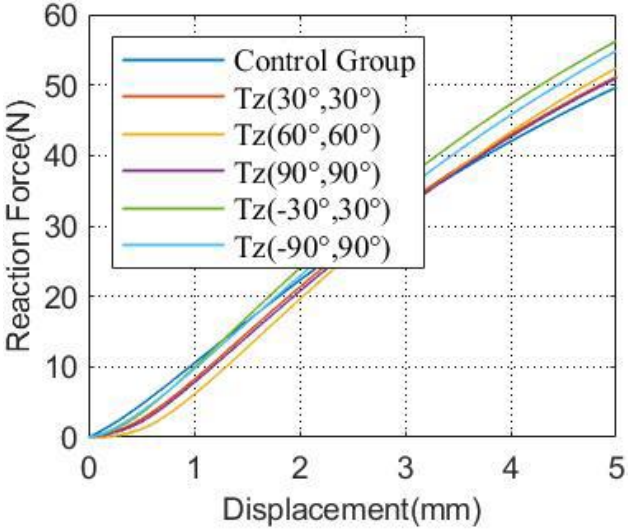
h_c = 5 cm,
d_s =3.2 mm,
Cellular size: $2 \times 2 \times 2$
Material:TPU

Compression – X: Perpendicular to print direction

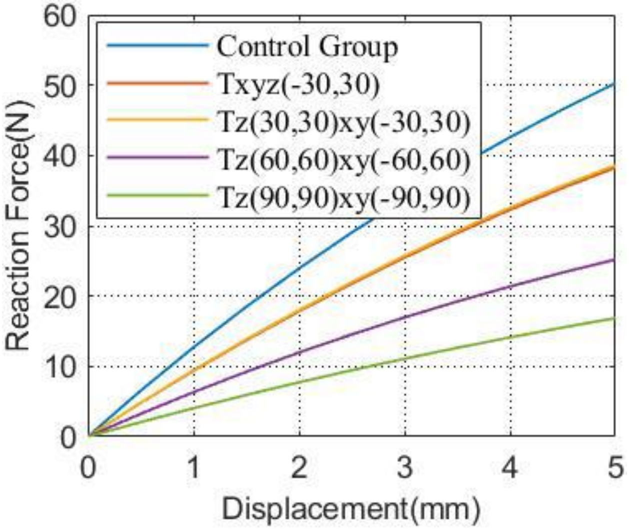
Simulations | **Experiments**

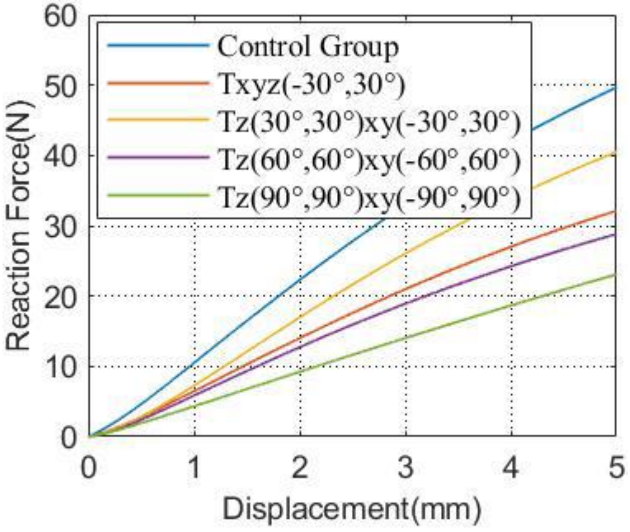
Twist Z faces





Twist Z, Y, X faces



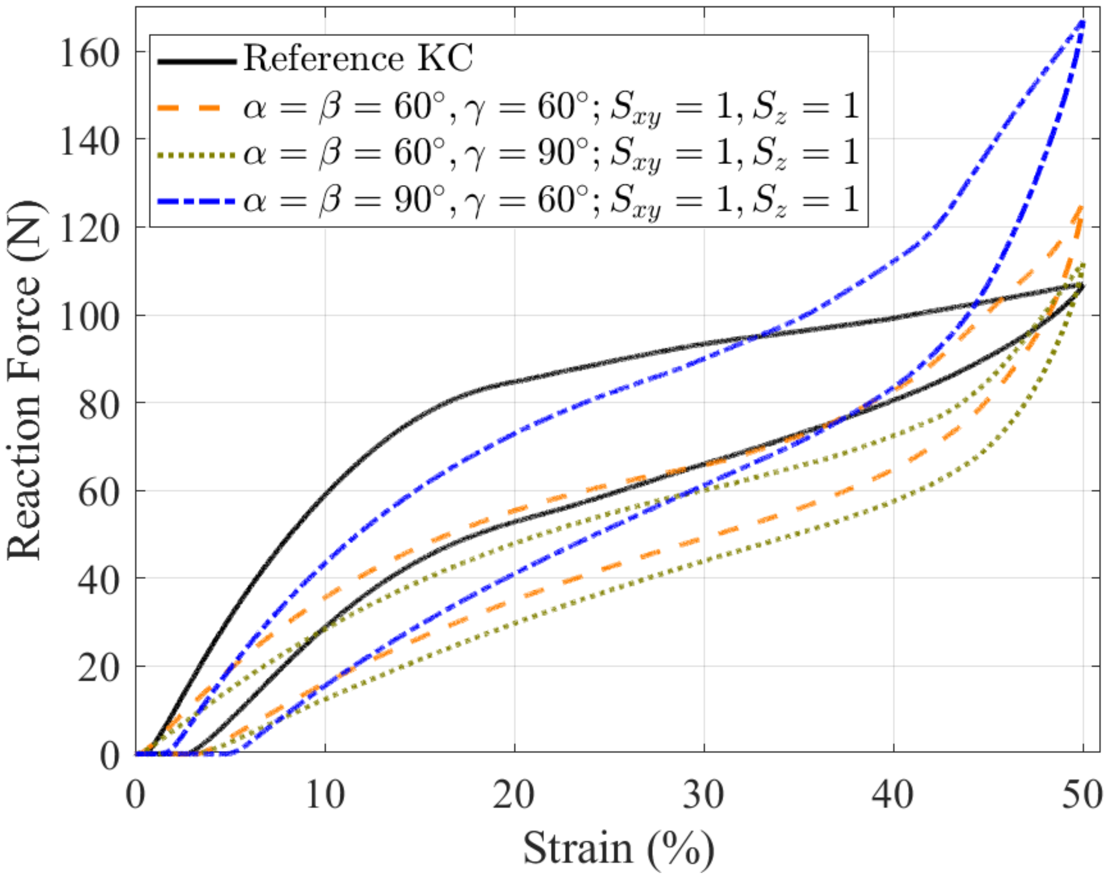


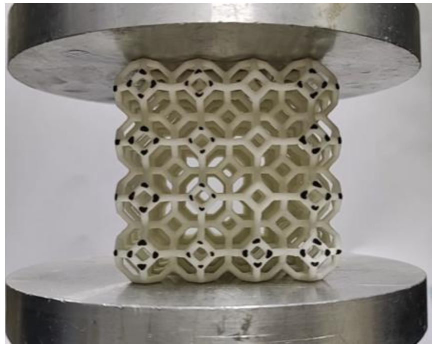
Relative error (%)

	X	Z
KC	1.09	1.06
$T_z(30°, 30°)$	5.03	6.59
$T_z(60°, 60°)$	9.74	6.17
$T_z(90°, 90°)$	0.79	0.57
$T_z(-30°, 30°)$	15.4	0.99
$T_z(-90°, 90°)$	8.73	1.95
$T_{xyz}(-30°, 30°$	16.08	16.08
$T_z(30°, 30°)$ $T_{xy}(-30°, 30°)$	4.92	3.86
$T_z(60°, 60°)$ $T_{xy}(-60°, 60°)$	14.29	5.88
$T_z(90°, 90°)$ $T_{xy}(-90°, 90°)$	37.09	21.23

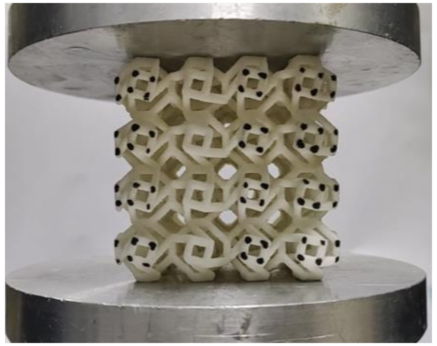
Compression Tests: Z - direction

Cell size (mm)	Diameter (mm)	Twist X, Y ($\alpha = \beta$)	Twist Z (γ)	Stretch X,Y	Stretch Z
14	2	$60^\circ/90^\circ$	$60^\circ/90^\circ$	1	1

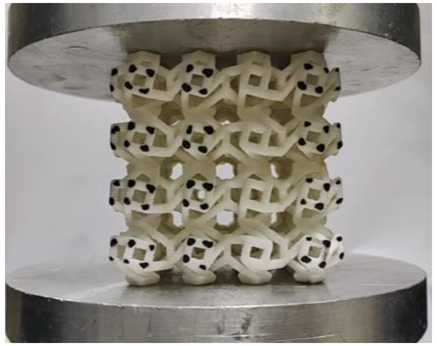




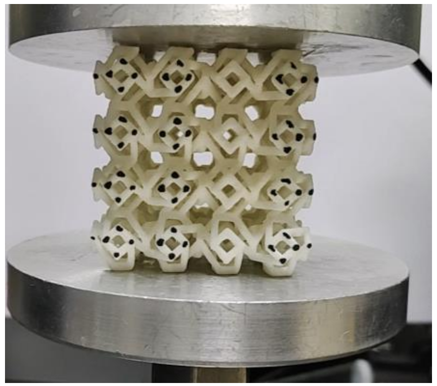
Reference Kelvin cell (KC)



$\alpha = \beta = 60^\circ, \gamma = 60^\circ$



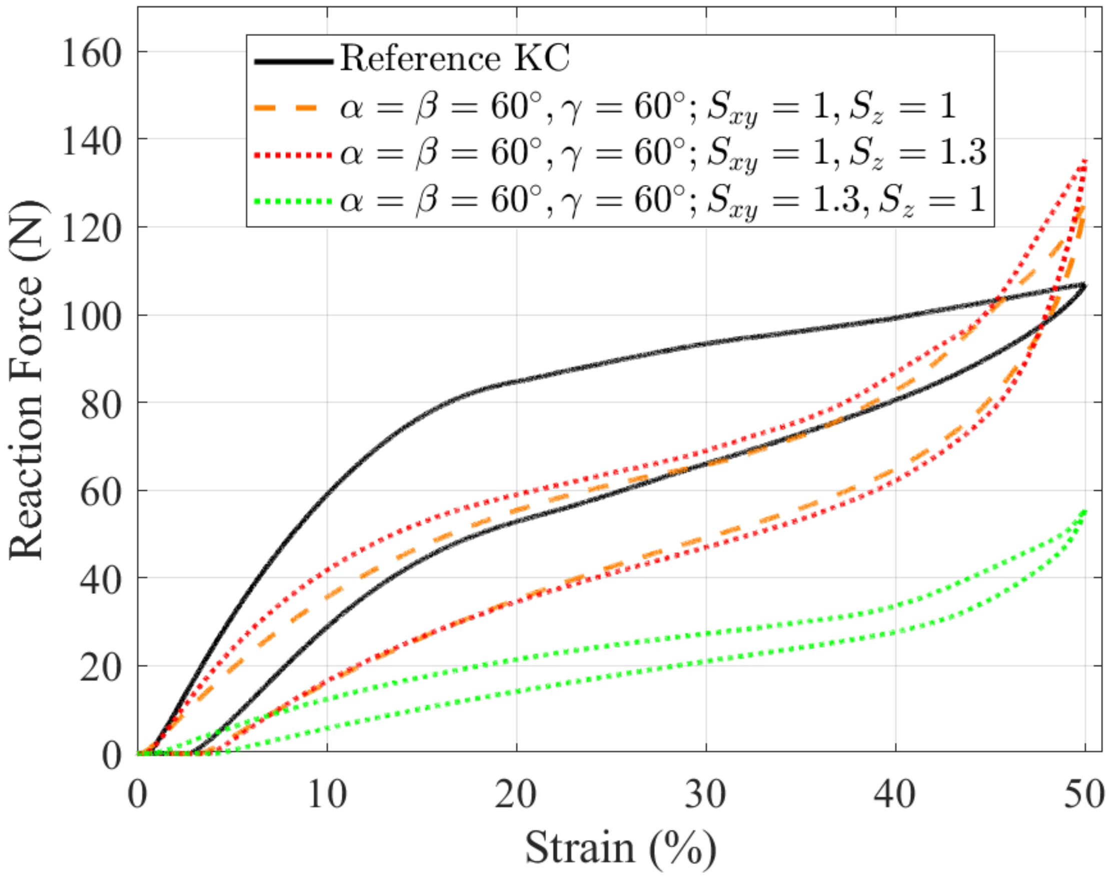
$\alpha = \beta = 60^\circ, \gamma = 90^\circ$

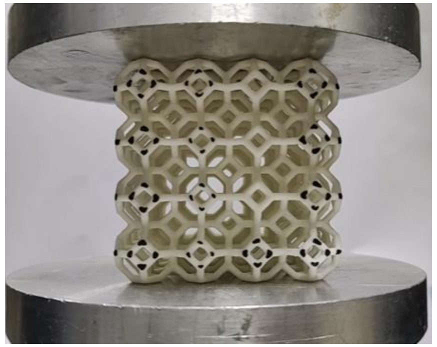


$\alpha = \beta = 90^\circ, \gamma = 60^\circ$

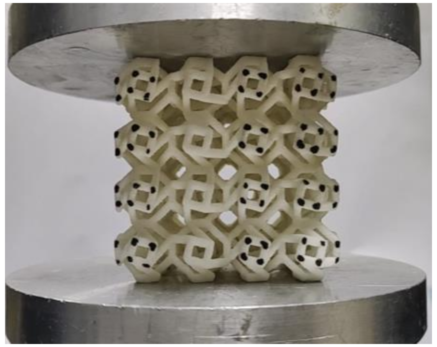
Compression Tests: W/O Stretch

Cell size (mm)	Diameter (mm)	Twist X, Y ($\alpha = \beta$)	Twist Z (γ)	Stretch S_{xy}	Stretch S_z
14	2	60°	60°	1/1.3	1/1.3

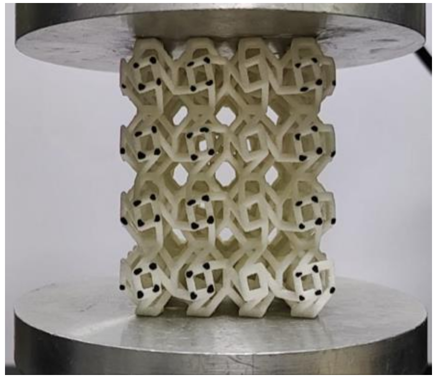




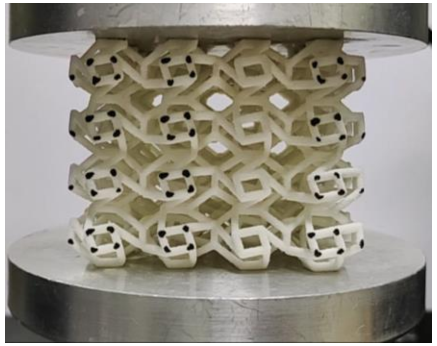
Reference Kelvin cell (KC)



$S_{xy} = 1, S_z = 1$
$\alpha = \beta = \gamma = 60^\circ$

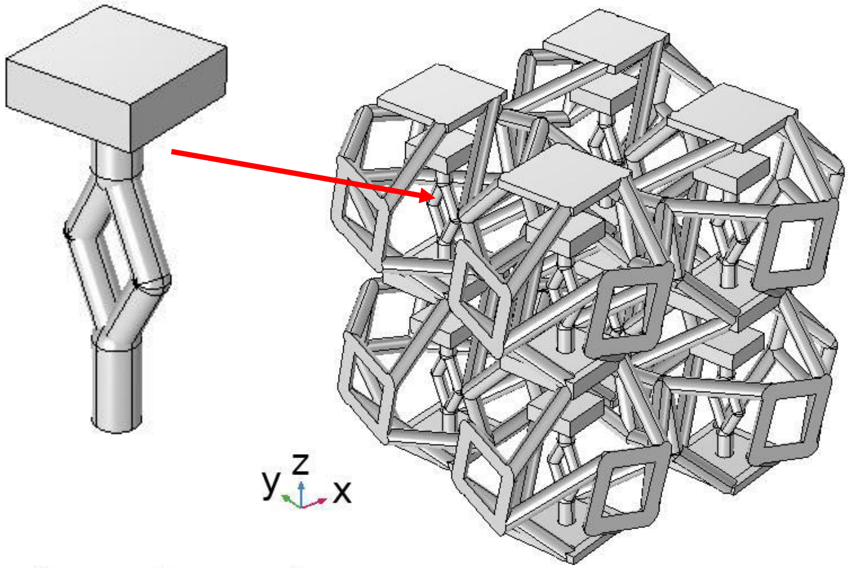


$S_{xy} = 1, S_z = 1.3$
$\alpha = \beta = \gamma = 60^\circ$

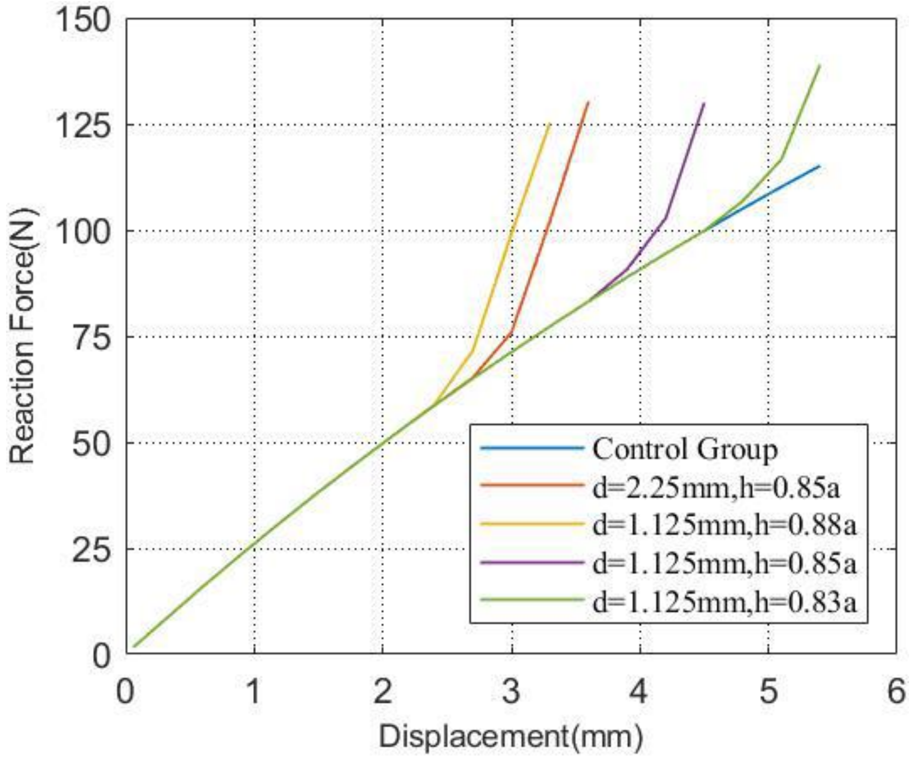


$S_{xy} = 1.3, S_z = 1$
$\alpha = \beta = \gamma = 60^\circ$

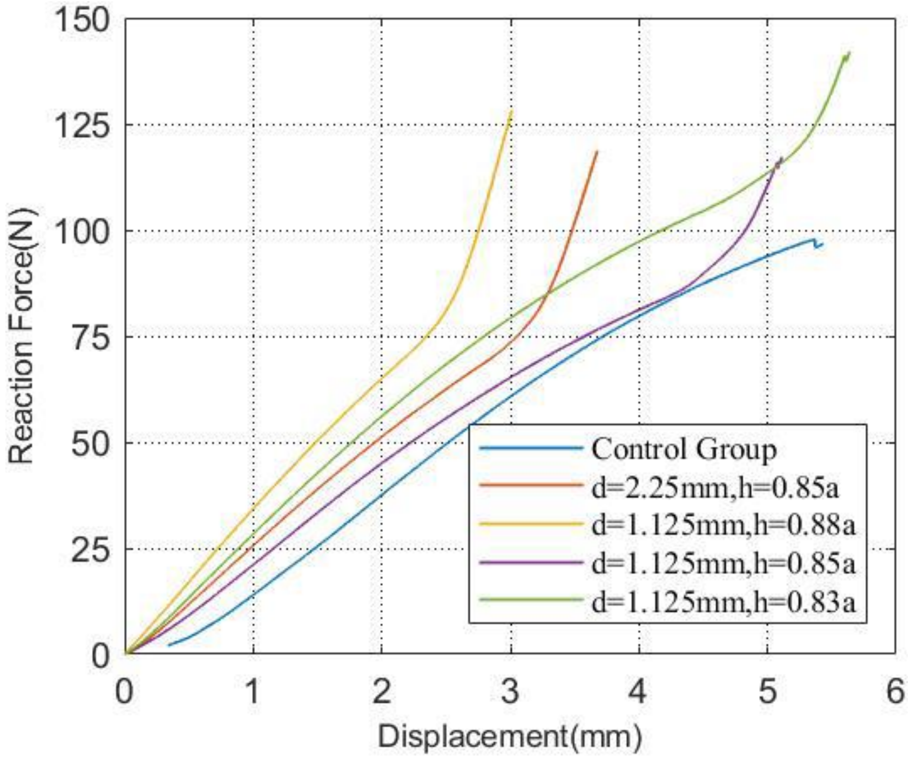
Add reinforced structures

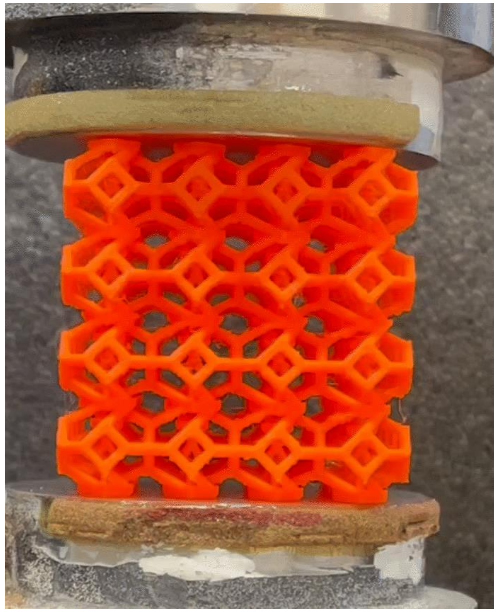


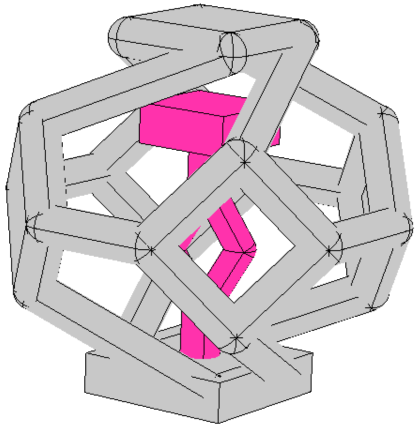
Simulations

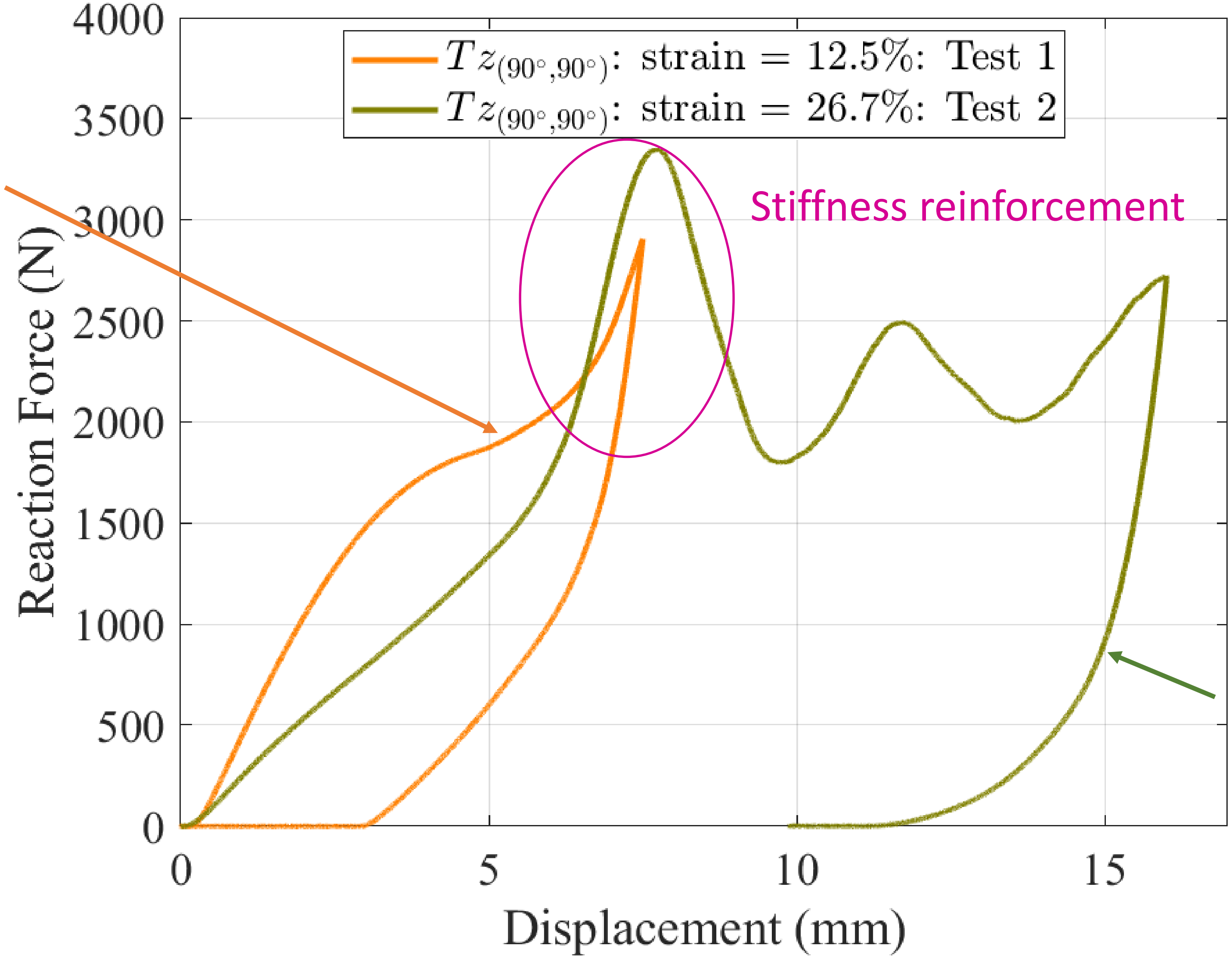


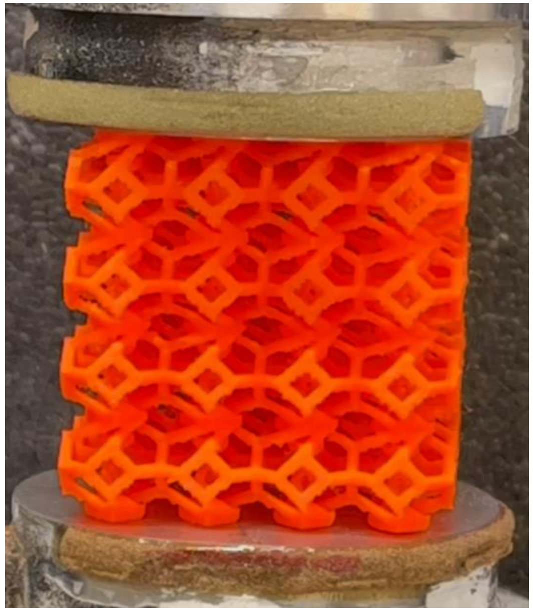
Experiments











Cell size: 1.5 cm
Beam radius: 0.9 mm
Material: PLA
AM: PDM
Printing direction: Z
Cell: Twist Z of 90°

Part IV: 3D Auxetic + Anisotropic

Classical Linear Anisotropic Elasticity

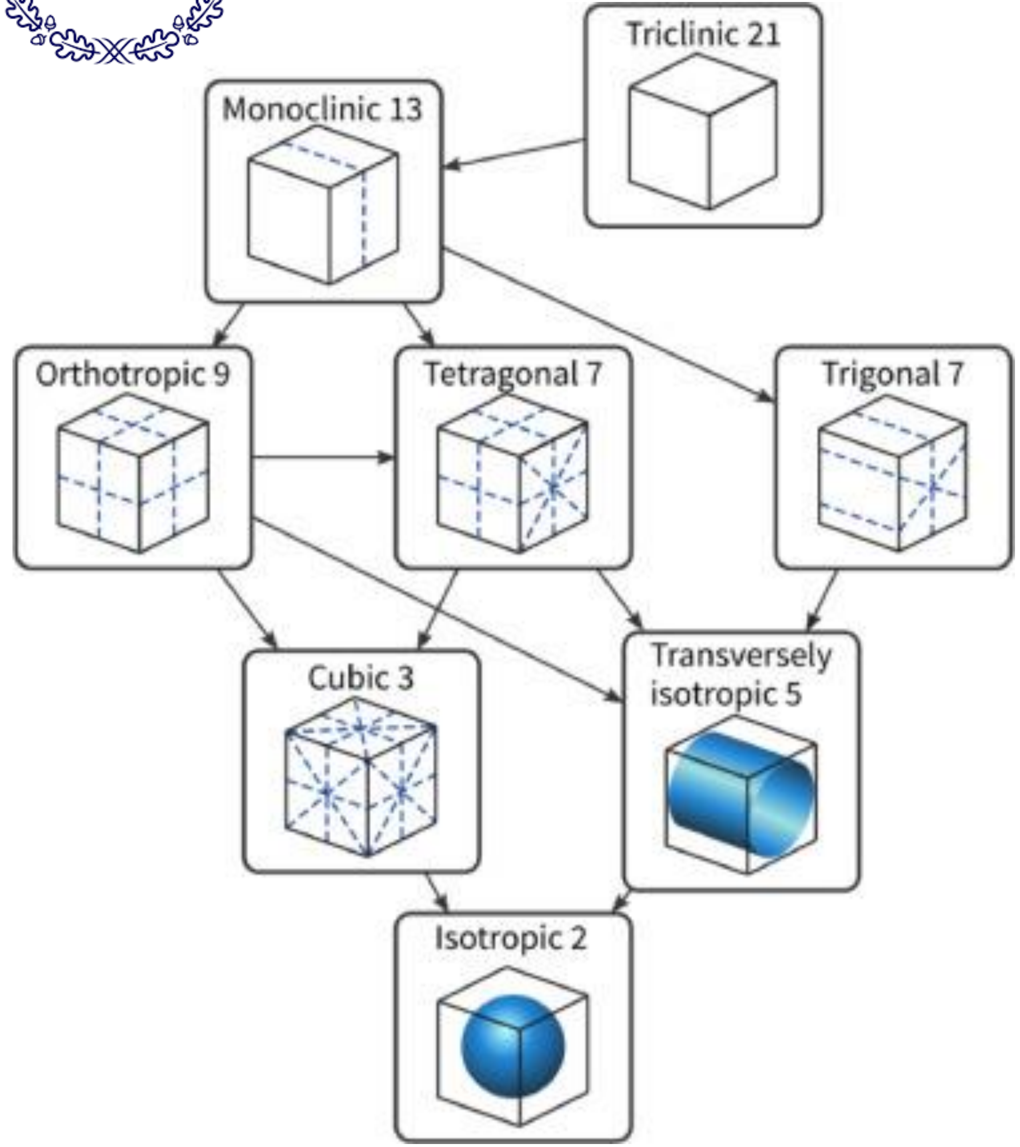
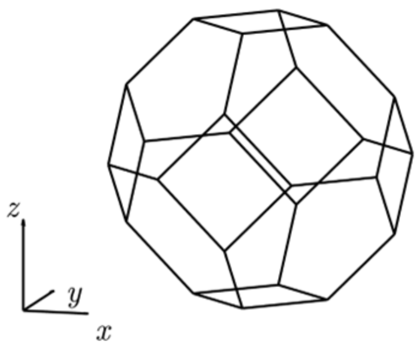


Table 5. The distinct symmetries of linear anisotropic elasticity.

Type of Material Symmetry	Number of Planes of Mirror Symmetry	Number of Planes of Isotropy	Number of Independent Elastic Coefficients	Restrictions on the Elastic Coefficients in the Reference Coordinate System for that Symmetry
Triclinic	0	0	18 (21)	None
Monoclinic	1	0	12 (13)	$\hat{c}_{61} = \hat{c}_{51} = \hat{c}_{52} = \hat{c}_{62} = \hat{c}_{64} = \hat{c}_{54} = \hat{c}_{53} = \hat{c}_{63} = 0$, $\hat{c}_{56} = 0$ or $\hat{c}_{41} = 0$.
Orthotropic or Orthorhombic	3	0	9	All of the conditions for monoclinic plus $\hat{c}_{41} = \hat{c}_{42} = \hat{c}_{43} = \hat{c}_{56} = 0$.
Tetragonal	5	0	6 (7)	All of the conditions for orthorhombic plus $\hat{c}_{11} = \hat{c}_{22}$, $\hat{c}_{23} = \hat{c}_{13}$, $\hat{c}_{55} = \hat{c}_{44}$
Cubic	9	0	3	All of the conditions for tetragonal plus $\hat{c}_{11} = \hat{c}_{33}$, $\hat{c}_{12} = \hat{c}_{13}$, $\hat{c}_{44} = \hat{c}_{66}$.
Trigonal	3	0	6 (7)	$\hat{c}_{43} = \hat{c}_{53} = \hat{c}_{63} = \hat{c}_{61} = \hat{c}_{54} = \hat{c}_{62} = 0$, $\hat{c}_{11} = \hat{c}_{22}$, $\hat{c}_{13} = \hat{c}_{23}$, $-\hat{c}_{42} = \hat{c}_{56} = \hat{c}_{14}$, $\hat{c}_{44} = \hat{c}_{55}$, $-\hat{c}_{15} = \hat{c}_{25} = \hat{c}_{46} = 0$ $\hat{c}_{66} = \hat{c}_{11} - \hat{c}_{22}$.
Hexagonal	7	0	5	All of the conditions for trigonal plus $-\hat{c}_{42} = \hat{c}_{56} = \hat{c}_{14}$.
Transverse Isotropy	$1 + \infty^1$	1	5	All of the conditions for trigonal plus $-\hat{c}_{42} = \hat{c}_{56} = \hat{c}_{14}$.
Isotropy	∞^2	∞^2	2	All of the conditions for cubic plus $\hat{c}_{44} = \hat{c}_{11} - \hat{c}_{22}$.

3D Full Anisotropic Cell + Auxetic

Cubic Kelvin cell (KC)

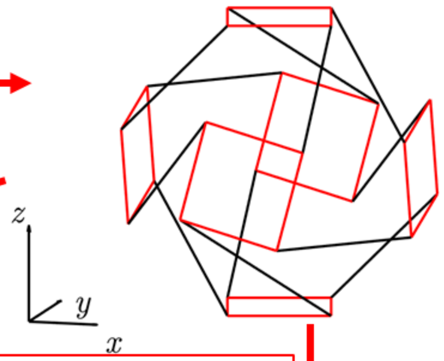


Antisymmetrical

Twist all faces of KC of a same angle

Geometry symmetry

New Kelvin cell



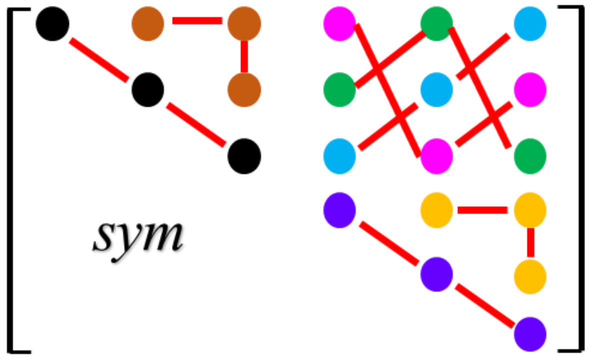
Chirality:

1 Rotational Symmetry

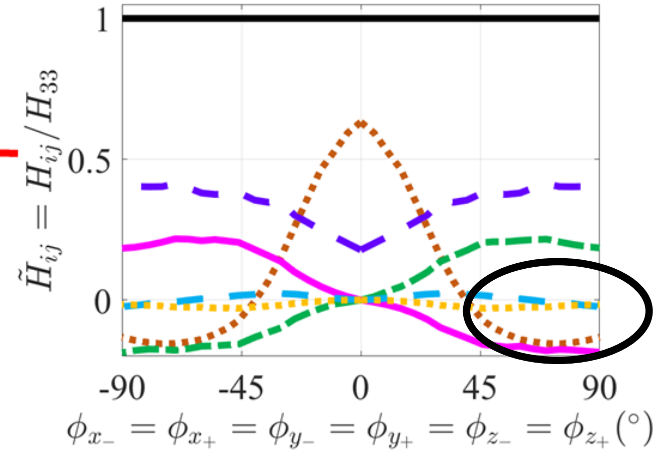
+

0 Mirror Face

Inverse characterization



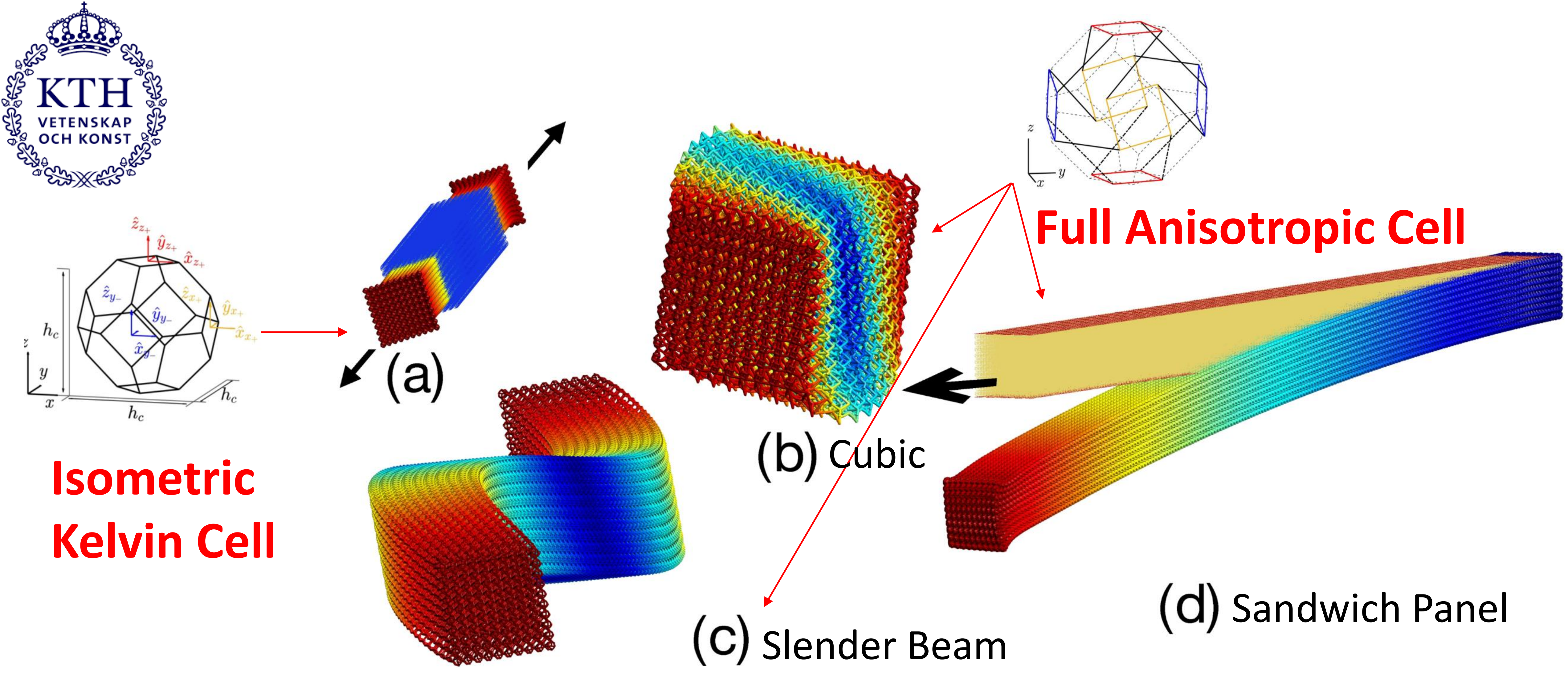
Symbolic representation of Hooke's tensors: **New rotational symmetry around axis $e_x + e_y + e_z$ of $\mp 120°$**



Anisotropic Hook's tensor

Auxetic

Negative Poisson's effect

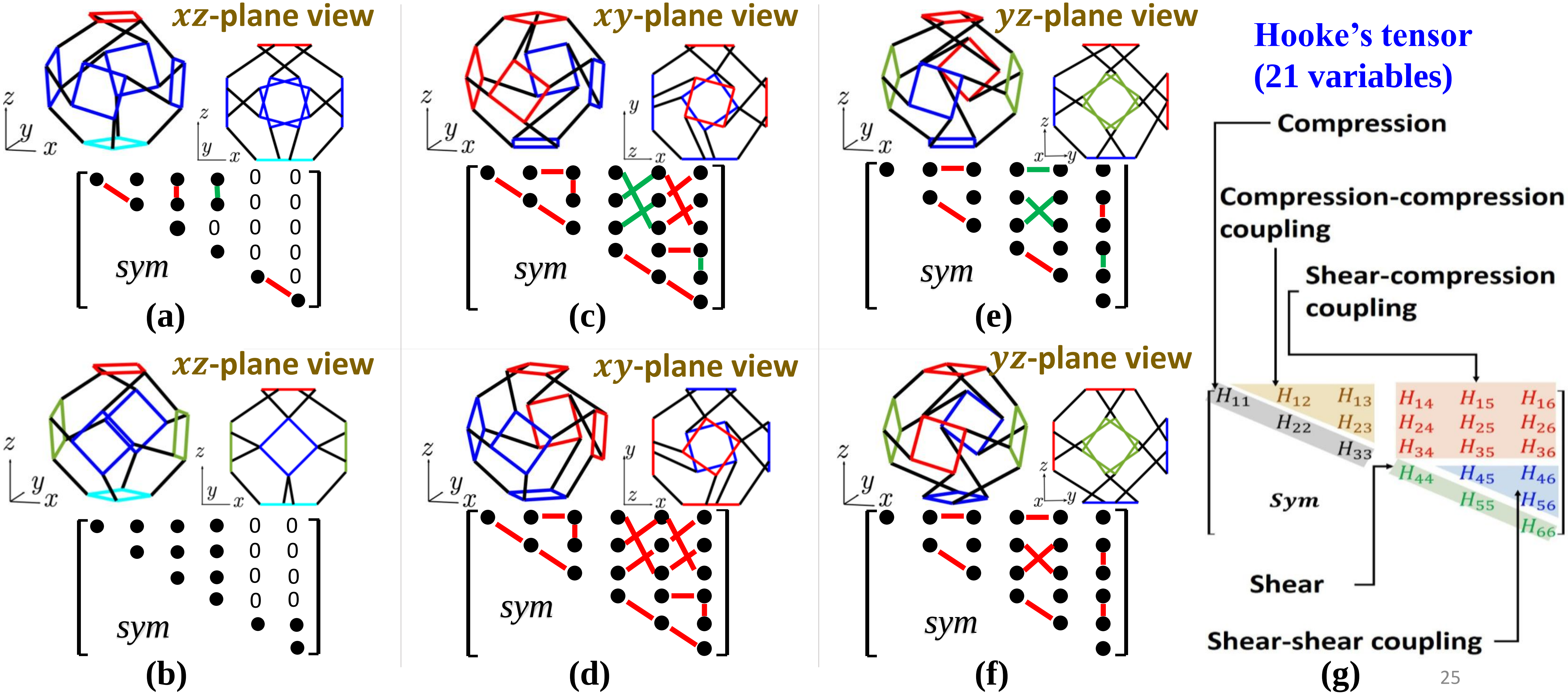


Uniaxial Extension： a)， Isometric Array， b) - d): Anisotropic lattice arrays

Uniaxial extension: anisotropic lattice arrays with different shapes vs an isometric cell array. a) reference isometric Kelvin cell array (undeformed mesh in blue, transverse displacements constrained at both ends); b) cubic sample, anisotropic cell; c) slender sample, anisotropic cell, length to edge ratio 20; d) cantilever beam sandwich with anisotropic cell core.

Distorted Kelvin Cells & New Anisotropic Hooke's Matrix

Examples: Has Rotational Symmetry but NO Mirror Face



Part V: Are they useful?

Potential Applications

- Energy absorption: vibration, shock force, bending, indentation resistance, etc.
- Multi-scale vibro-acoustic structures.
- Large coupling deformation applications: soft robotics, sensors.
- Thermal insulation: Multi-material lattices.
- Lightweight multifunctional applications for aerospace: ceramic, metal 3D printed lattice structures.

Conclusion

- A new group of 3D auxetic lattices and extended to full anisotropy.
- Discover new linear anisotropic elasticity without mirror plan those not belong to classical linear anisotropic types.
- Programmable Poisson's effect and elastic coupling behaviors.
- Inverse characterized the anisotropic elastic material properties.
- Primary study of 3D printing anisotropic lattices.

Thanks!

huina@kth.se

Symbolic Expression of Linear Anisotropic Elasticity