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Effect of Pore Size Polydispersity on Acoustic Properties of High-Porosity Solid Foams

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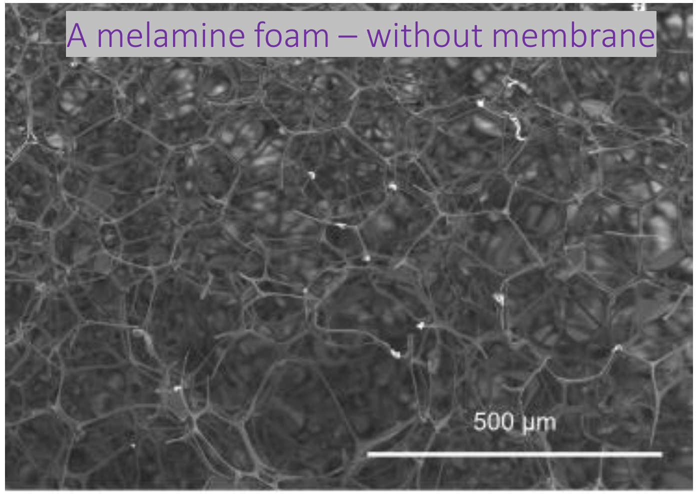
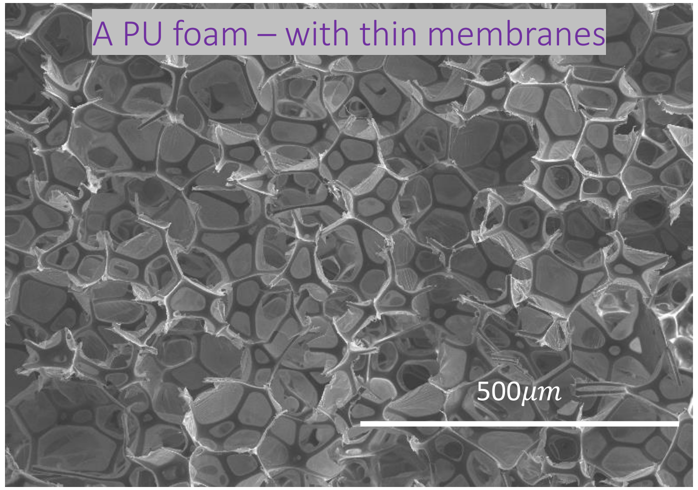
Introduction & objectives

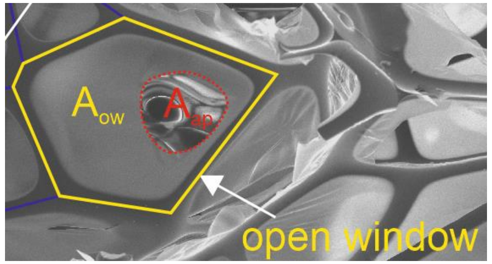
- Real foams
  - Acoustic or thermal insulation
  - Without or with membranes (very thin, $t_m \approx 0.3\mu m$)
  - Local heterogeneity in pore sizes
  - High porosity ($\phi \approx 1$)
- Acoustic properties
  - Transport parameters/Sound absorption coefficient
  - Resulting from microstructure features
  - Simulation (at pore-scale): very expensive (generally)
- Objectives
  - Study the effect of pore size polydispersity on transport parameters of random foams
  - Predict acoustic properties from microstructural parameters (pore size distribution, mean aperture ratio,…)

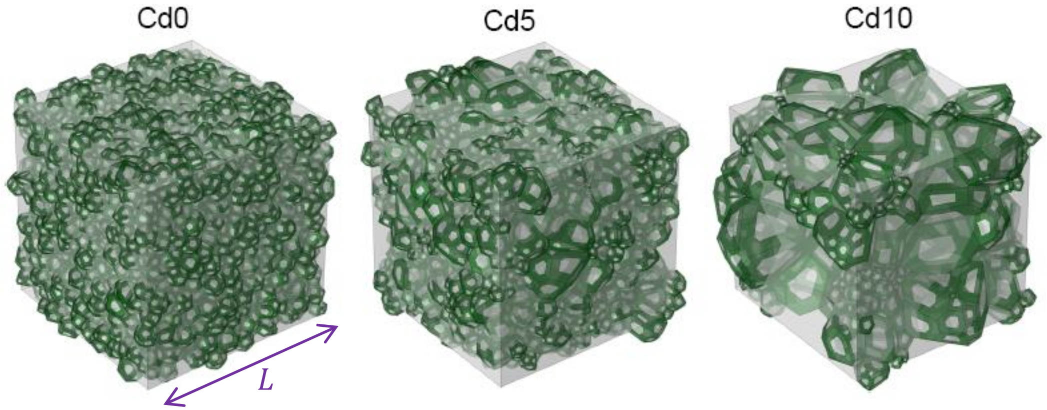

A melamine foam – without membrane


A PU foam – with thin membranes

Microstructure reconstruction

- Periodic unit cell (PUC) is generated by an adaptive Laguerre tessellation[1]
- Log-normal distribution of pore sizes
- Pore size polydispersity (Coefficient of variation):
  - $C_d = \frac{\sigma_d}{d}$
  - d: Average pore size; σ_d: Standard deviation.
- Aperture ratio of open windows (membranes)
  - $\tau_o = \sqrt{\frac{A_{ap}}{A_{ow}}}$





Samples of PUC microstructures. The PUC size is normalized by pore size average, $L/d = 10$.

Name	Cd0	Cd1	Cd2	Cd3	Cd4	Cd5	Cd6	Cd7	Cd8	Cd9	Cd10
C_d	0.030	0.107	0.198	0.285	0.366	0.440	0.511	0.574	0.646	0.689	0.730

Estimated polydispersity levels in the generated PUC microstructures

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[1]. [Quey et al., Comput. Method Appl. M. 330, 308 (2018)].

Transport parameters simulation

- Static viscous permeability k_0:

$$k_0 = \mu \frac{\langle \mathbf{v} \rangle \cdot \mathbf{e}}{\lambda_m} \qquad \begin{aligned} \mu \Delta \mathbf{v} - \nabla p &= -\nabla p \quad \text{in} \quad \Omega_f \,, \\ \nabla . \mathbf{v} &= 0 \quad \text{in} \quad \Omega_f \,, \\ \mathbf{v} &= 0 \quad \text{on} \quad \partial\Omega \,, \end{aligned}$$

- High-frequency tortuosity α_∞ and Viscous characteristic length Λ:

$$\Lambda = \frac{2 \int_{\Omega_f} \mathbf{E}^2 dV}{\int_{\partial\Omega} \mathbf{E}^2 dS} \,, \qquad \alpha_\infty = \frac{\langle \mathbf{E}^2 \rangle}{\langle \mathbf{E} \rangle^2} \qquad \begin{aligned} \nabla \cdot \mathbf{E} &= 0 \quad \text{in} \quad \Omega_f \,, \\ \mathbf{E} &= -\nabla \varphi + \mathbf{e} \quad \text{in} \quad \Omega_f \,, \\ \mathbf{E} \cdot \mathbf{n} &= 0 \quad \text{on} \quad \partial\Omega \,, \end{aligned}$$

- Static thermal permeability k'_0:

$$k'_0 = \phi \langle u \rangle \,, \qquad \begin{aligned} \Delta u &= -1 \quad \text{in} \quad \Omega_f \,, \\ u &= 0 \quad \text{on} \quad \partial\Omega \,. \end{aligned}$$

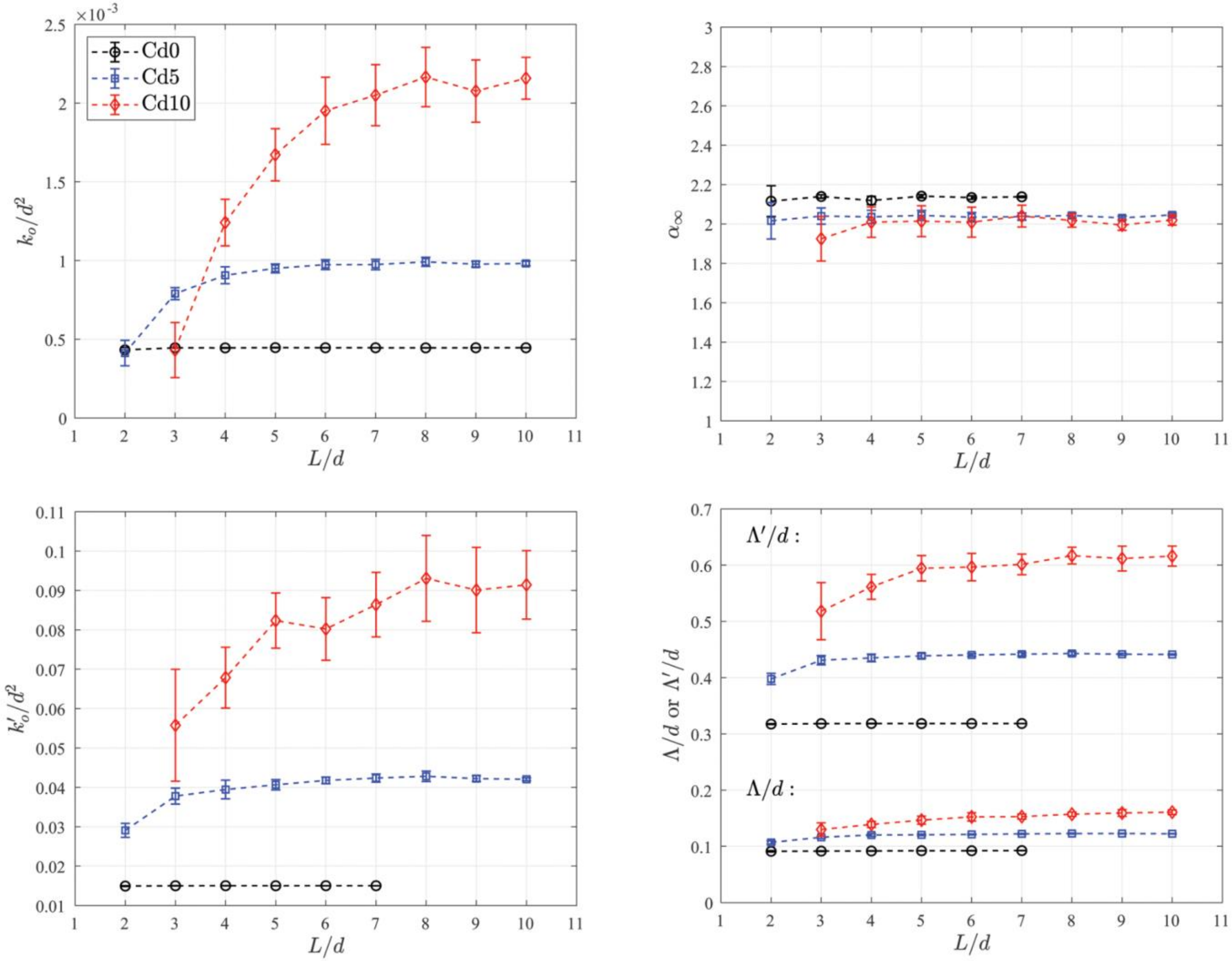
- Thermal characteristic length Λ':

$$\Lambda' = 2 \int_{\Omega_f} dV \Big/ \int_{\partial\Omega} dS \,,$$

- Open porosity
  - $\phi = 1$
- Normalized transport parameters (by the average pore size d):
  - $k_0^* = \frac{k_0}{d^2}, \; {k'_0}^* = \frac{k'_0}{d^2}$
  - $\Lambda^* = \frac{\Lambda}{d}, \quad \Lambda'^* = \frac{\Lambda'}{d}$
  - $\alpha_\infty^* = \alpha_\infty$
- Normalized size of the periodic unit cell (PUC):
  - L/d

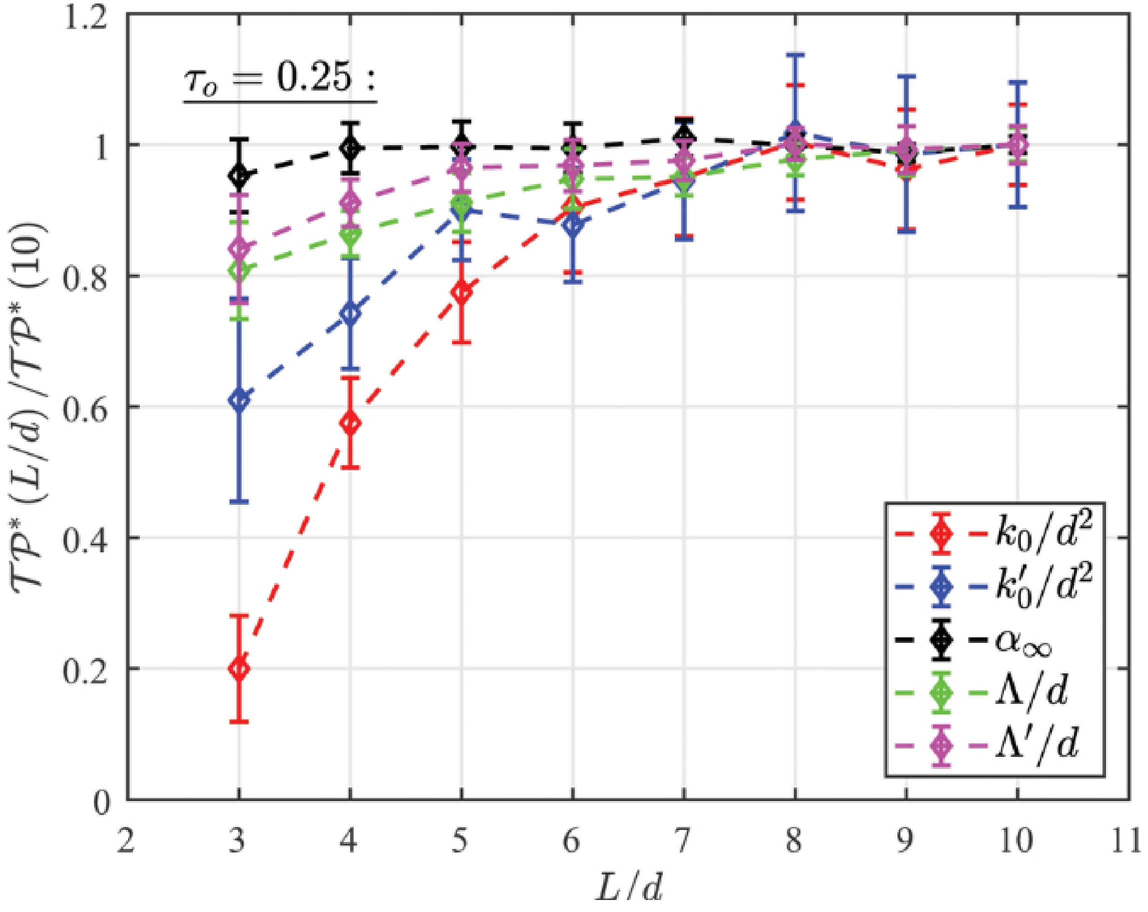
On the size of the periodic unit cell

- All transport parameters tend to converge as the size of the PUC increases.
- PUC size converged → Representative elementary volume (REV)
- The convergence depends on the pore size dispersion:
  - The greater the polydispersity, the larger the REV size.



On the size of the periodic unit cell

- The convergence on the PUC size is not the same for all transport parameters:
  - PUC needs a largest size to be a REV for k_0.
- Same observation for different τ_o
- Message for experimental characterization:
  - A sample size that achieves “convergence” for k_0 can be considered as reliable for characterizing the other transport parameters and acoustic properties.



Convergence of transport parameters for Cd10

Polydispersity effect and effective pore size

- Relating transport parameters of a polydisperse foam to a reference monodisperse one:

$$\gamma = \left(\frac{\mathcal{TP}^{*}(C_d)}{\mathcal{TP}^{*}_{mono}} \right)^{1/\beta},$$

  - $\beta = 1$: Λ, Λ' and α_∞
  - $\beta = 2$: k_0 and k_0'

- Observations:
  - α_∞ almost unchanged
  - $k_0, k_0', \Lambda, \Lambda'$ increase with C_d
  - γ do not depend on τ_o (but only on C_d)

- Simulate a polydisperse foam by an equivalent monodisperse foam, what is the pore size ?

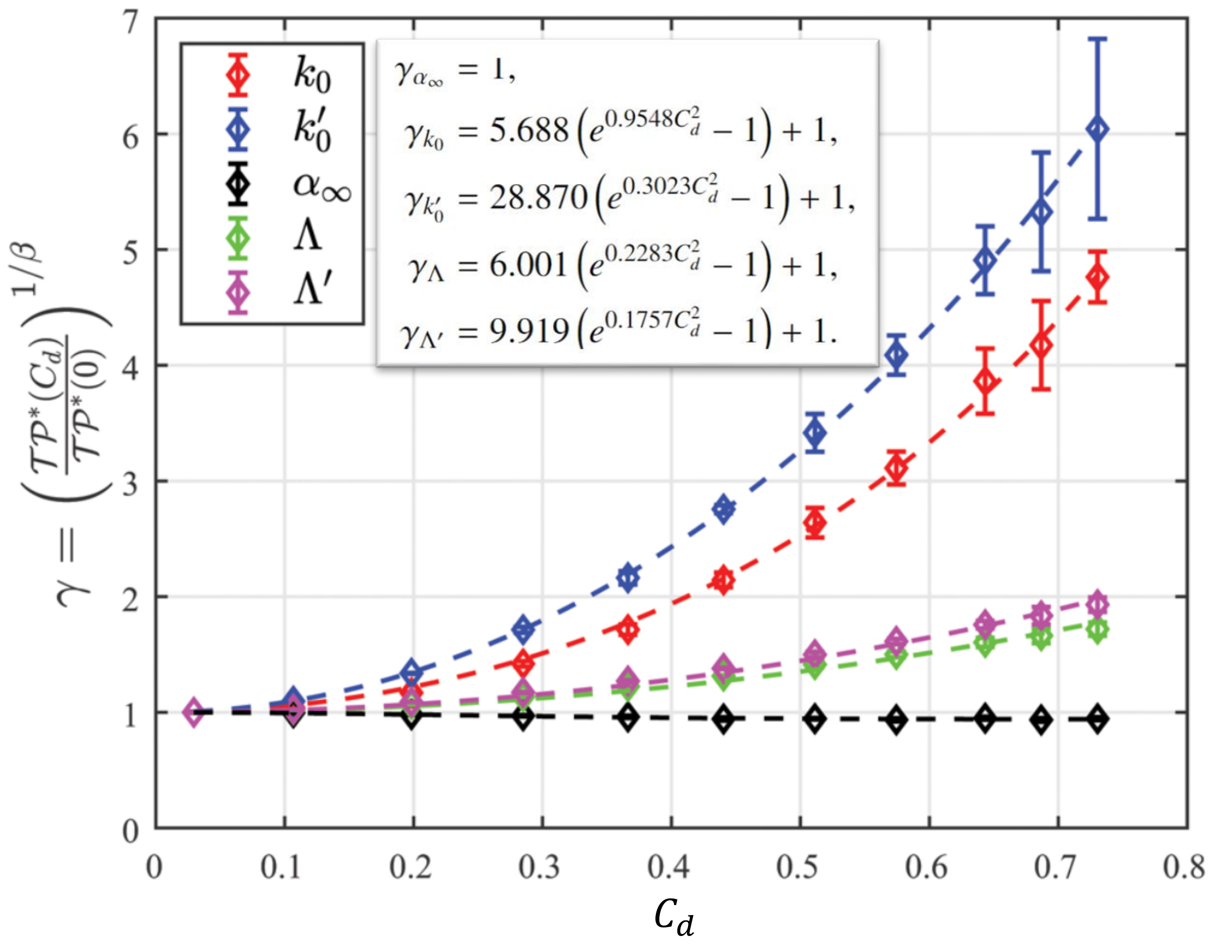
$$D = d \times \gamma$$

  - For each transport parameter, there exists an effective pore size:

$$D_{k_0'} \geq D_{k_0} \geq D_{\Lambda'} \geq D_{\Lambda} \geq D_{\alpha_\infty} = d$$

  - Monodisperse foam:

$$D_{k_0'} = D_{k_0} = D_{\Lambda'} = D_{\Lambda} = D_{\alpha_\infty} = d$$



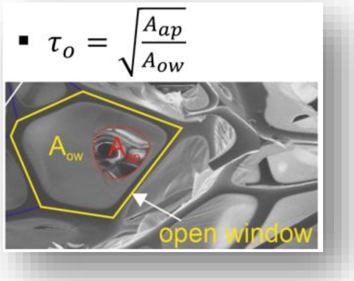
Surrogate models for transport parameters

- Transport parameter of a random polydisperse foam:

$$\mathcal{TP}^*\left(C_d\right) = (\gamma)^{\beta}\,\mathcal{TP}^*(0)\,,$$

- γ is calculated from C_d
- Normalized transport parameters of reference monodisperse foam depends on τ_o (Kelvin-cell simulation):
    - With membranes, $0.1 \leq \tau_o \leq 0.9$:

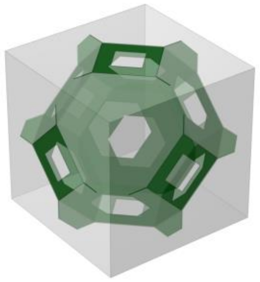
$$\begin{aligned}
\alpha_\infty^*(0) &= 497.88\tau_o^6 - 1691.9\tau_o^5 + 2322.2\tau_o^4 \\
&\quad - 1649.9\tau_o^3 + 646.21\tau_o^2 - 136.69\tau_o + 14.334, \\
k_0^*(0) &= 0.0283\tau_o^3, \\
k_0'^*(0) &= 0.0994\tau_o^3 - 0.0735\tau_o^2 + 0.0242\tau_o + 0.0122, \\
\Lambda^*(0) &= 31.357\tau_o^5 - 65.618\tau_o^4 + 51.741\tau_o^3 \\
&\quad - 18.44\tau_o^2 + 3.1726\tau_o - 0.1573, \\
\Lambda'^*(0) &= 44.585\tau_o^5 - 93.24\tau_o^4 + 73.568\tau_o^3 \\
&\quad - 26.054\tau_o^2 + 4.1708\tau_o + 0.0704.
\end{aligned}$$

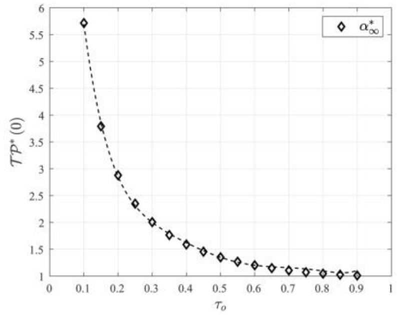


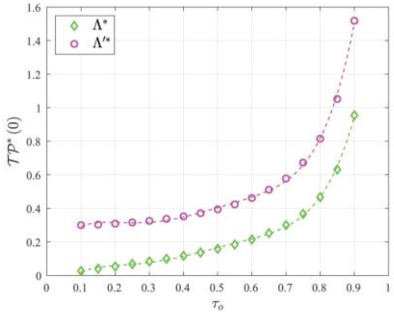
- $\tau_o = \sqrt{\frac{A_{ap}}{A_{ow}}}$

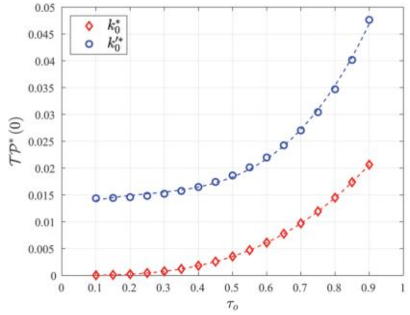
    - Without membrane, $\tau_o \approx 1$

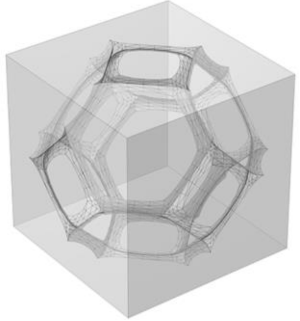
ϕ	$k_0^*(0)$	$k_0'^*(0)$	$\Lambda^*(0)$	$\Lambda'^*(0)$	$\alpha_\infty^*(0)$
0.986	0.018	0.038	0.623	1.078	1.03



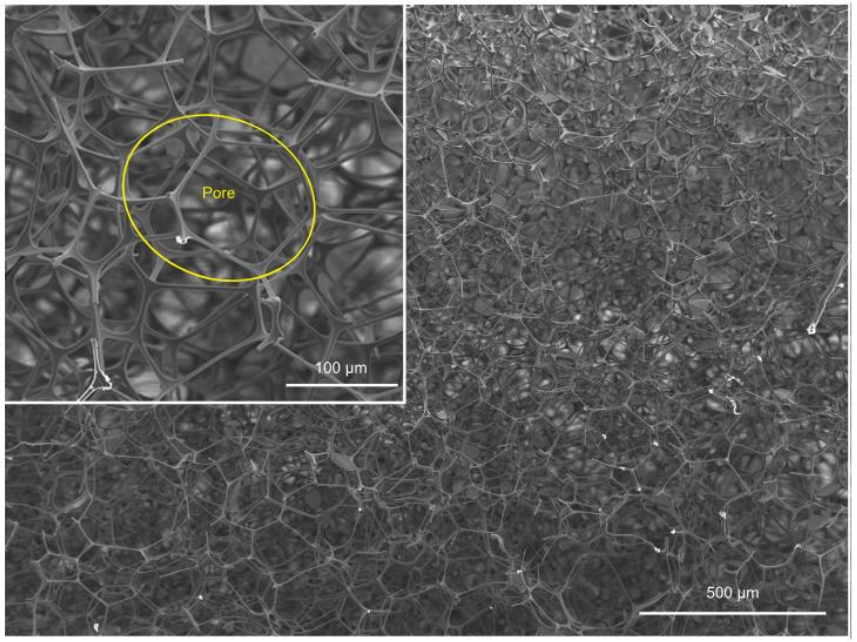




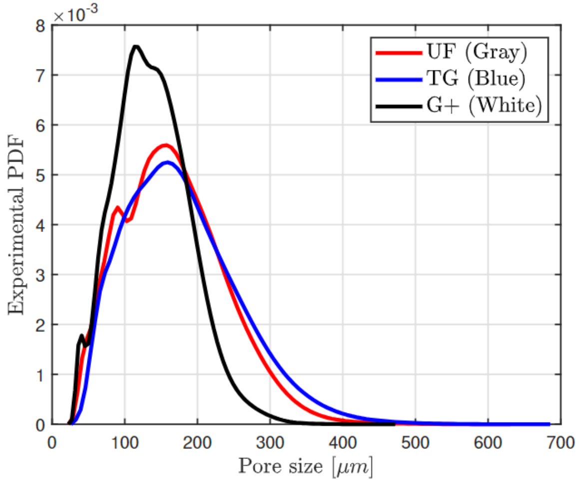


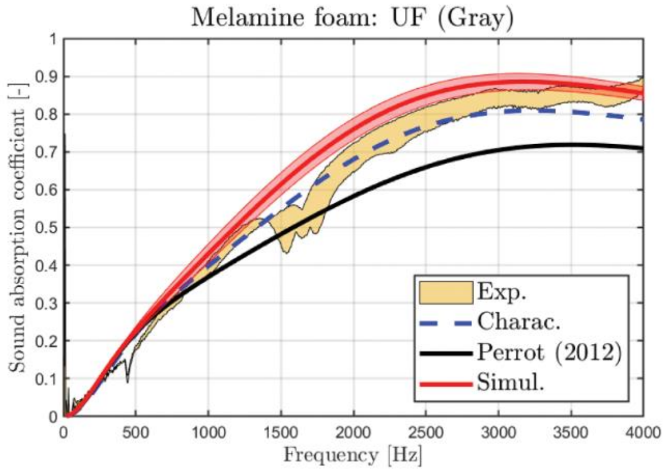


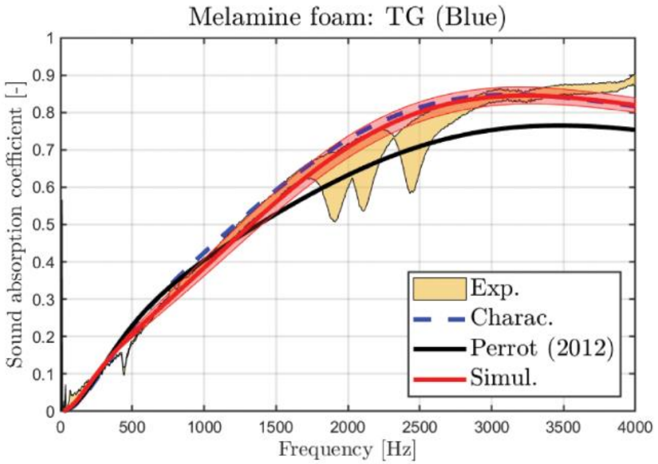
Application on open-cell melamine foams

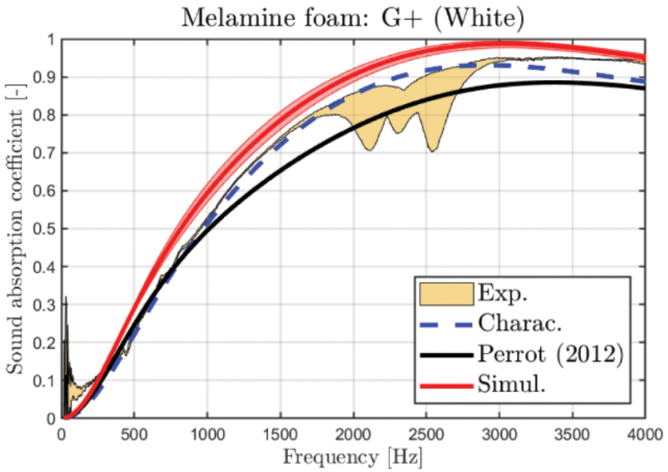


Pore size estimation of a melamine foam sample









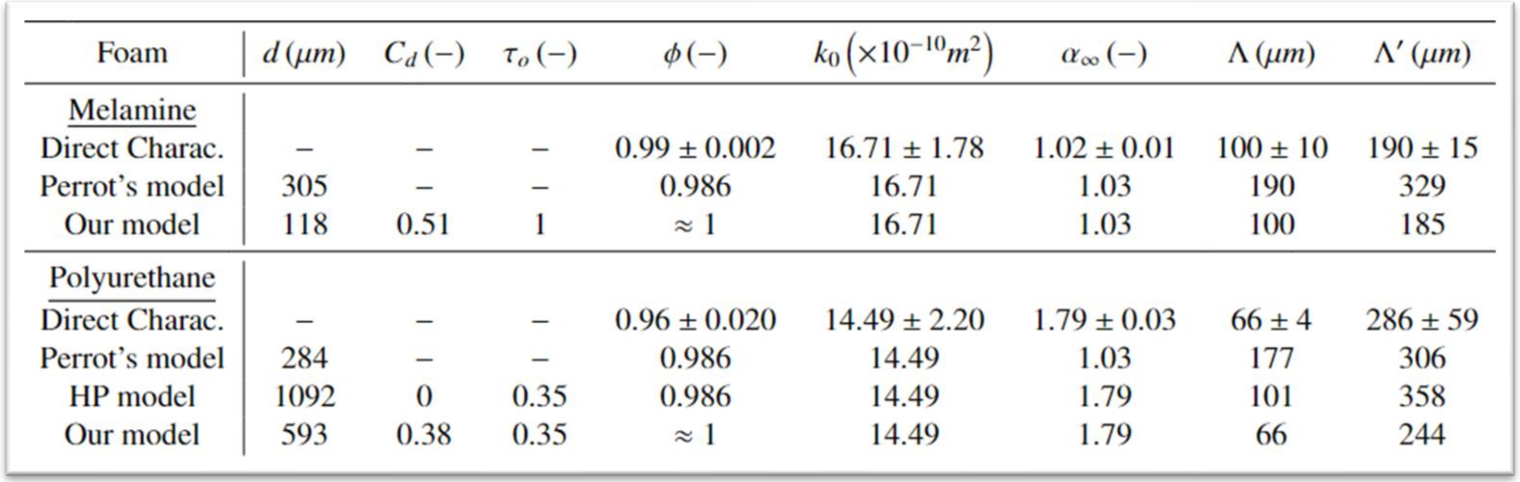
- Note: In Perrot *et al* model (2012)[1], the polydispersity is not considered.

[1] [Perrot et al., J. Appl. Phys. **111**, 014911 (2012)].

Inverse identification of the morphological features

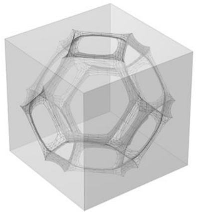
- Transport parameters are provided (measurements, $TP(C_d)$) → Information about microstructure of <u>high-porosity</u> foams (C_d, d, τ_o)

- $\{\phi \approx 1, \alpha_\infty, k_0, \Lambda, \Lambda'\}$
- $\alpha^*_\infty(0) = \alpha_\infty \to \tau_o$
- $\tau_o \to \begin{Bmatrix} k_0^*(0) \\ \Lambda^*(0) \\ \Lambda'^*(0) \end{Bmatrix}$
- $\begin{Bmatrix} k_0/k_0^*(0) \\ \Lambda/\Lambda^*(0) \end{Bmatrix} \to \{C_d, d\}$
- Verify: $\{C_d, d, \tau_o\} \to \Lambda'$ vs Λ'

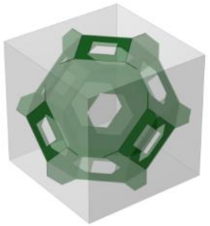


Foam	$d\,(\mu m)$	$C_d\,(-)$	$\tau_o\,(-)$	$\phi(-)$	$k_0\left(\times 10^{-10} m^2\right)$	$\alpha_\infty\,(-)$	$\Lambda\,(\mu m)$	$\Lambda'\,(\mu m)$
<u>Melamine</u>								
Direct Charac.	–	–	–	0.99 ± 0.002	16.71 ± 1.78	1.02 ± 0.01	100 ± 10	190 ± 15
Perrot's model	305	–	–	0.986	16.71	1.03	190	329
Our model	118	0.51	1	≈ 1	16.71	1.03	100	185
<u>Polyurethane</u>								
Direct Charac.	–	–	–	0.96 ± 0.020	14.49 ± 2.20	1.79 ± 0.03	66 ± 4	286 ± 59
Perrot's model	284	–	–	0.986	14.49	1.03	177	306
HP model	1092	0	0.35	0.986	14.49	1.79	101	358
Our model	593	0.38	0.35	≈ 1	14.49	1.79	66	244

- In **Perrot et al model**[1], foams are considered as <u>monodispersed without membranes</u>, d is calculated from k_0, the remaining parameters ($\alpha_\infty, \Lambda, \Lambda'$) are simulated from a Kevin-cell with d

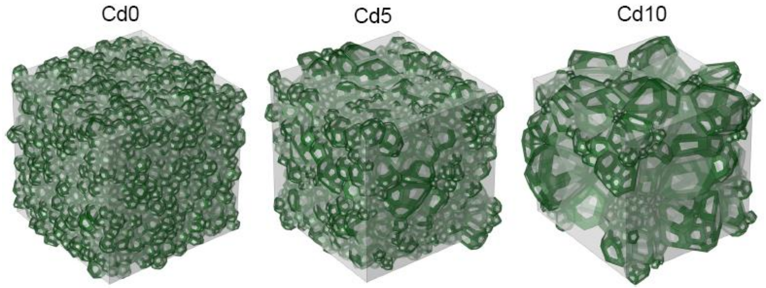


- In **HP model**[2], foams are considered as <u>monodispersed with membranes</u>, d and τ_o are calculated from k_0 and α_∞, the remaining parameters (Λ, Λ') are then simulated from a Kelvin-cell with d and τ_0



1. [Perrot et al., J. Appl. Phys. **111**, 014911 (2012)]; 2. [Hoang and Perrot. J. Appl. Phys. **112**, 054911 (2012)]

Conclusions



- The convergence of the REV size is not the same for all transport parameters:
  - Depending on the foam's polydispersity C_d
  - REV size needs to be larger for the viscous permeability (than the other parameters)

- Effect of pore size polydispersity (with d, τ_o constant):
  - Keeping α_∞ unchanged
  - Increasing $k_0, k'_0, \Lambda, \Lambda'$

- Surrogate models are proposed:
  - Direct calculation of the acoustical properties from the corresponding microstructural descriptors of foam (C_d, d, τ_o)
  - Identifying morphological properties through the inverse analysis of measured transport parameters.

Thanks for your attention!

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