Acoustic characterization of anisotropic porous materials

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Motivation

- Current laboratory techniques are **not able to produce reproducible and reliable pore parameters**
- Porous absorbents are assumed to be acoustically isotropic
- Porous materials possess **a marked anisotropy**, which influences their acoustical behavior
- A general method for the characterization of anisotropic fluid materials is needed

Equivalent fluid model: isotropic vs. anisotropic material

Acoustical parameters: *R*, *T*

2 parameters

→ Scattering matrix →

Equivalent fluid bulk properties:

Isotropic (2 parameters): *ρ*, *B*

Anisotropic (7 parameters): ***ρ***, *B*

→ JCAL model →

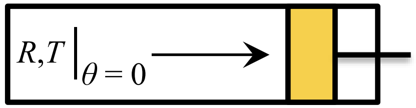
Non-acoustical (pore) parameters

Isotropic: 6 parameters

Anisotropic: 12 parameters

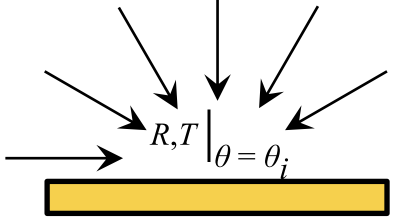
- Tortuosity (x3: 3 principal directions)
- Viscous characteristic length (x3)
- Static viscous permeability (x3)
- Open porosity
- Thermal characteristic length
- Static thermal permeability

Isotropic medium



Measurement at normal incidence (1 angle)

Anisotropic medium

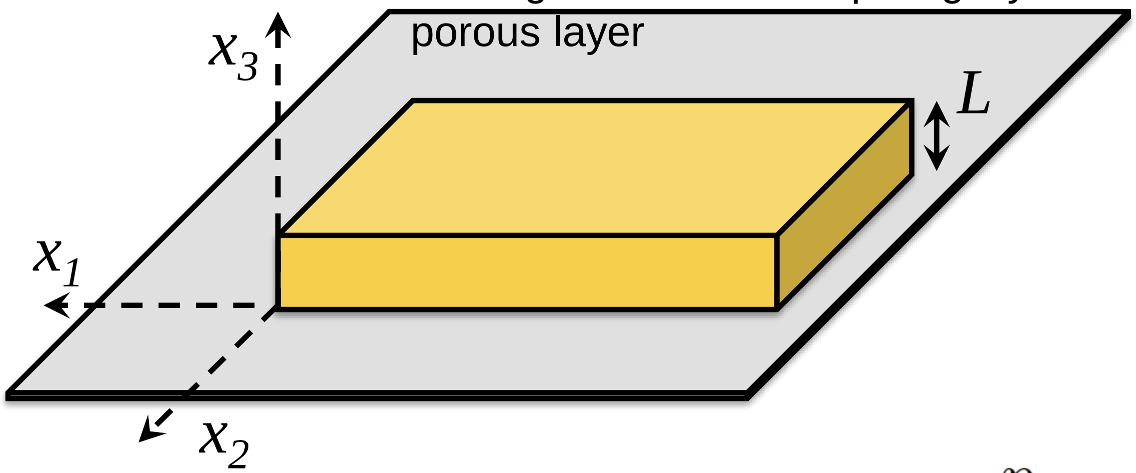


Measurement at oblique incidence (6 angles)

Literature

- Few studies have attempted to extend inversion procedures to characterize anisotropic materials
- Terroir et al. (2019) have shown that the bulk modulus and density tensor of a layer of homogeneous anisotropic material surrounded by air on both sides can be inferred from knowledge of the reflection and transmission coefficients at six arbitrary angles of incidence
- Morin et al. (Forum Acusticum 2023): experimental validation in an impedance tube
- This study:
  - Extends to the case of a rigidly-backed porous layer: reflection coefficients only
  - Propose a free-field experimental method to deduce the bulk modulus and the density tensor
  - Benchmark for the impedance tube measurements and vice versa

Formulation of the problem



Symmetric density tensor

Mass and momentum conservation:

$$j\omega \frac{p_l}{B} = \nabla \cdot \boldsymbol{v}_l, \qquad j\omega \boldsymbol{\rho} \cdot \boldsymbol{v}_l = \nabla p_l$$

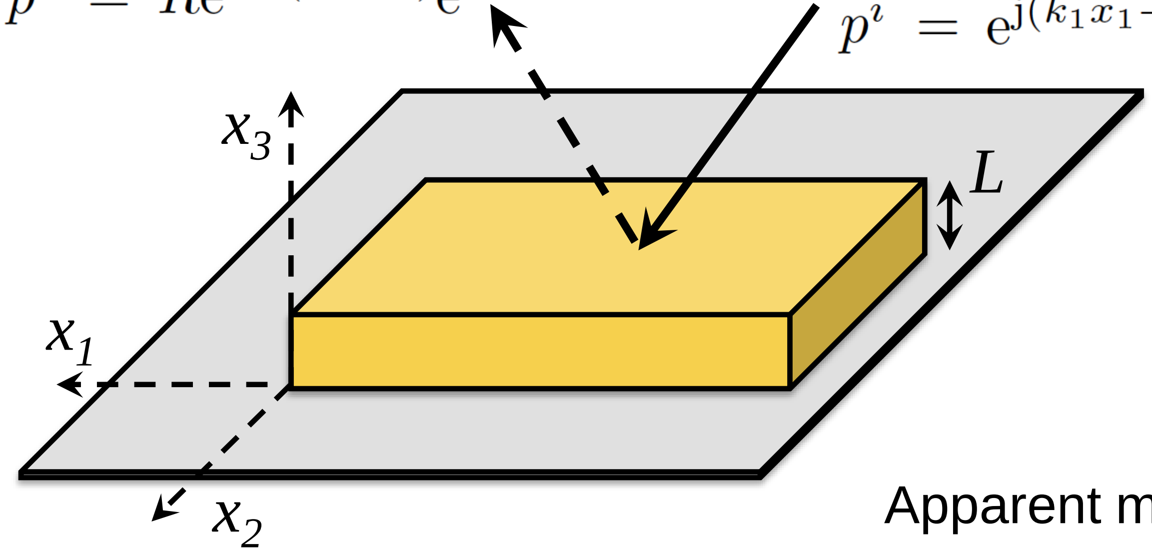
$$j\omega \frac{p_a}{B_0} = \nabla \cdot \boldsymbol{v}_a, \qquad j\omega \rho_0 \cdot \boldsymbol{v}_a = \nabla p_a$$

Nolan et al., 2023

Formulation of the problem

$$p^r = R\mathrm{e}^{\mathrm{j}k_3(x_3-L)}\mathrm{e}^{\mathrm{j}\boldsymbol{k}_\Gamma\cdot\boldsymbol{x}_\Gamma}$$

$$p^i = \mathrm{e}^{\mathrm{j}(k_1x_1+k_2x_2-k_3(x_3-L))}$$



$$p_l = \hat{p}_l(x_3)\mathrm{e}^{\mathrm{j}\boldsymbol{k}_\Gamma\cdot\boldsymbol{x}_\Gamma}$$

$$\boldsymbol{v}_l = \hat{\boldsymbol{v}}_l(x_3)\mathrm{e}^{\mathrm{j}\boldsymbol{k}_\Gamma\cdot\boldsymbol{x}_\Gamma}$$

$$\boldsymbol{k}_\Gamma = k_1\boldsymbol{e}_1 + k_2\boldsymbol{e}_2$$

$$\boldsymbol{x}_\Gamma = x_1\boldsymbol{e}_1 + x_2\boldsymbol{e}_2$$

Apparent mass and momentum conservation:

$$\mathrm{j}\omega\frac{\hat{p}_l}{\tilde{B}} = \mathrm{j}(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)\hat{v}_{l3} + \frac{\partial\hat{v}_{l3}}{\partial x_3}$$

$$\mathrm{j}\omega\tilde{\rho}\hat{v}_{l3} = \mathrm{j}(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)\hat{p}_l + \frac{\partial\hat{p}_l}{\partial x_3}$$

Nolan et al., 2023

Formulation of the problem

Apparent mass and momentum conservation:

$$\mathrm{j}\omega\frac{\hat{p}_l}{\tilde{B}} = \mathrm{j}(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)\hat{v}_{l3} + \frac{\partial \hat{v}_{l3}}{\partial x_3}$$

$$\mathrm{j}\omega\tilde{\rho}\hat{v}_{l3} = \mathrm{j}(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)\hat{p}_l + \frac{\partial \hat{p}_l}{\partial x_3}$$

Boundary conditions: $\hat{p}_l(0) = p_0, \qquad \hat{v}_{l3}(0) = 0$

$$\hat{p}_l(L) = 1 + R, \qquad \hat{v}_{l3}(L) = (R-1)/\tilde{Z}_0$$

Nolan et al., 2023

Apparent impedance and wavenumber

$$\tilde{Z} = \pm\tilde{Z}_0\sqrt{\frac{(R+1)^2}{(R-1)^2} - \frac{\left(p_0 e^{-j(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)L}\right)^2}{(R-1)^2}}$$

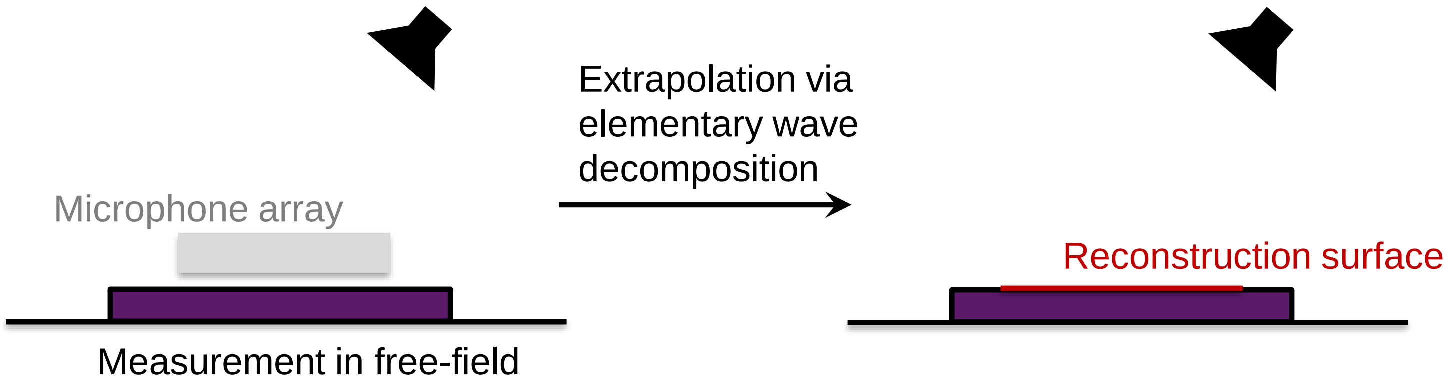
$$e^{\pm j\tilde{k}L} = \left[R\left(1 \pm \frac{\tilde{Z}}{\tilde{Z}_0}\right) + \left(1 \mp \frac{\tilde{Z}}{\tilde{Z}_0}\right)\right]\frac{1}{p_0 e^{-j(\boldsymbol{q}\cdot\boldsymbol{k}_\Gamma)L}}$$

$$\tilde{\rho} = \tilde{Z}\tilde{k}/\omega, \qquad \tilde{B} = \omega\tilde{Z}/\tilde{k}$$

Nolan et al., 2023

Measuring R at oblique incidence

- Richard et al. (2017), Nolan (2020)
- Angle-dependent surface impedance is measured via sound field reconstruction at the material's surface



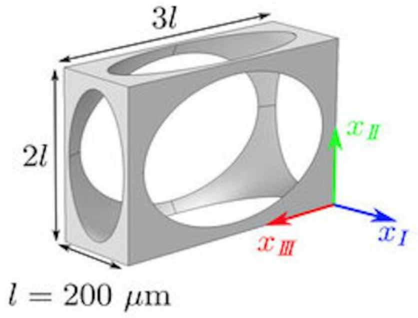
$$Z_S(\mathbf{r}_S) = -\frac{1}{\rho_0 c_0}\frac{\hat{p}(\mathbf{r}_S)}{\hat{v}_n(\mathbf{r}_S)} \qquad R = \frac{Z_S\cos(\theta) - 1}{Z_S\cos(\theta) + 1}$$

Simulated measurements on a synthetic anisotropic material

12 known pore parameters (multiple-scale method)

Direct →

Expected bulk modulus B and (inverse) density tensor coefficients H_{11}, H_{12}, H_{13}, H_{22}, H_{23}, H_{33} can be calculated



Terroir et al., 2019

$$\neq$$

Estimated bulk modulus B and (inverse) density tensor coefficients H_{11}, H_{12}, H_{13}, H_{22}, H_{23}, H_{33} can be calculated

Inverse ←

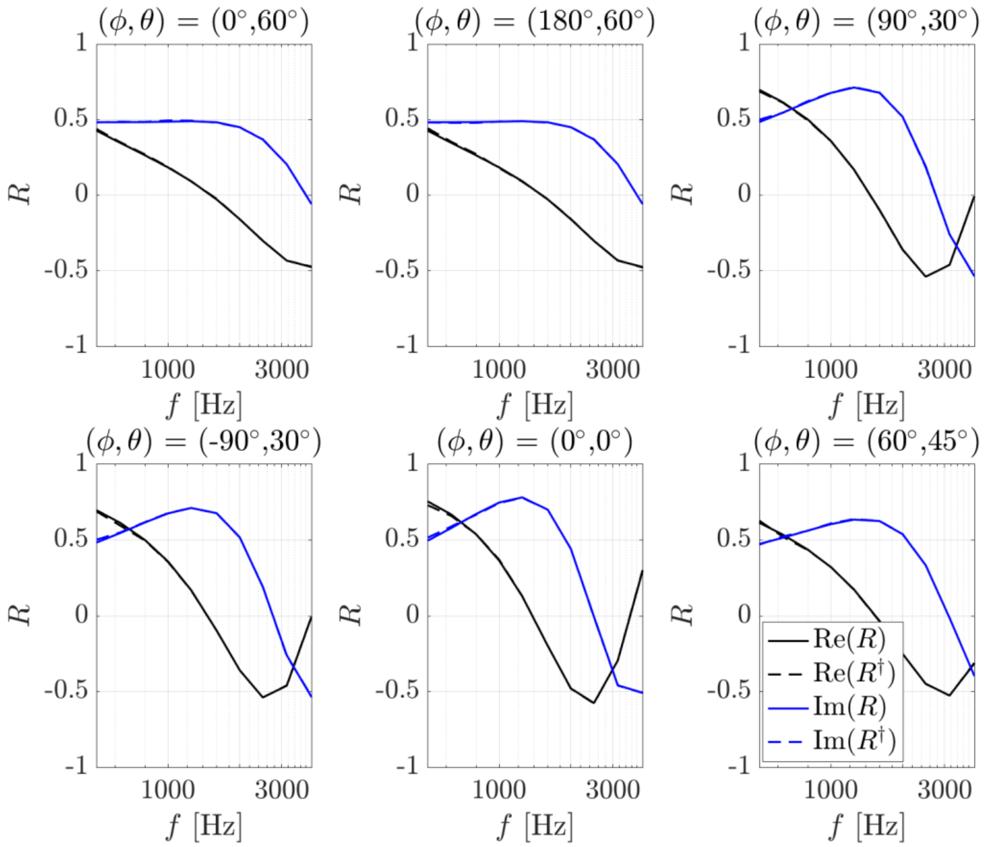
Measured reflection coefficients

Simulated measurements

- Array of 162 microphones distributed in two square layers
- Vertical/horizontal spacing of 3/2.5 cm, respectively
- Array aperture: 20 cm x 20 cm
- Distance to the material: 1.5 cm
- Sound field propagation: plane wave basis of 256 plane waves (uniformly distributed directions of propagation)
- 400 point-wise impedances are reconstructed at the material's surface on a grid of dimensions 10 cm x 10 cm
- Gaussian noise with SNR = 40 dB is added to the simulated pressure

Simulated measurements

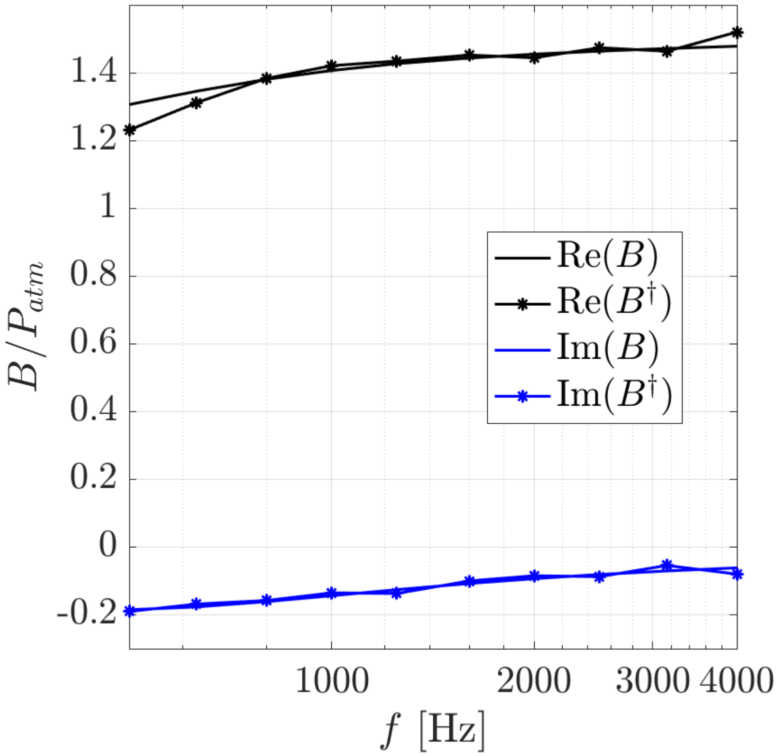
- Expected vs. measured reflection coefficients

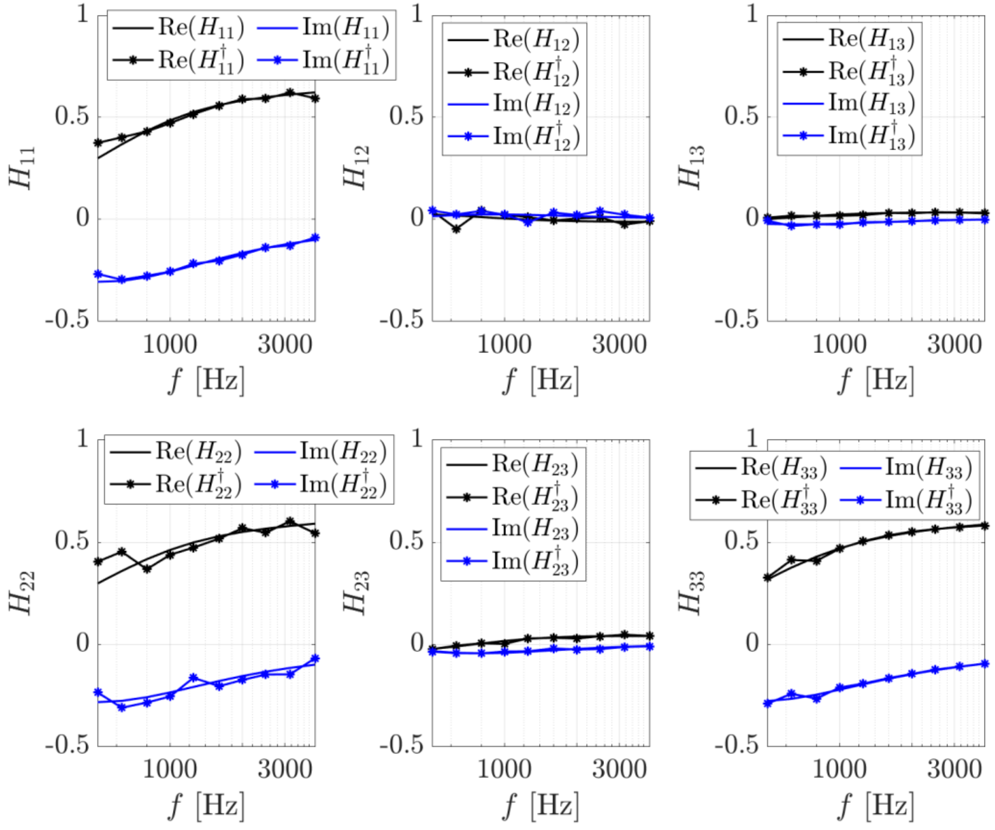


Nolan et al., 2023

Simulated measurements

- Expected vs. measured bulk modulus and density tensor coefficients





Nolan et al., 2023

Free-field measurements on a manufactured material

12 pore parameters estimated from R and T coefficients measured in an impedance tube

Direct →

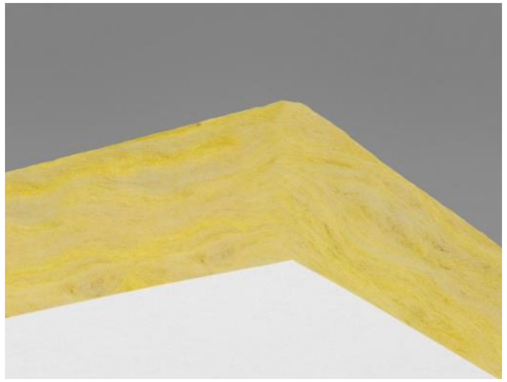
Expected bulk modulus B and (inverse) density tensor coefficients H_{11}, H_{12}, H_{13}, H_{22}, H_{23}, H_{33} can be calculated

$\neq$

Estimated bulk modulus B and (inverse) density tensor coefficients H_{11}, H_{12}, H_{13}, H_{22}, H_{23}, H_{33} can be calculated

← Inverse

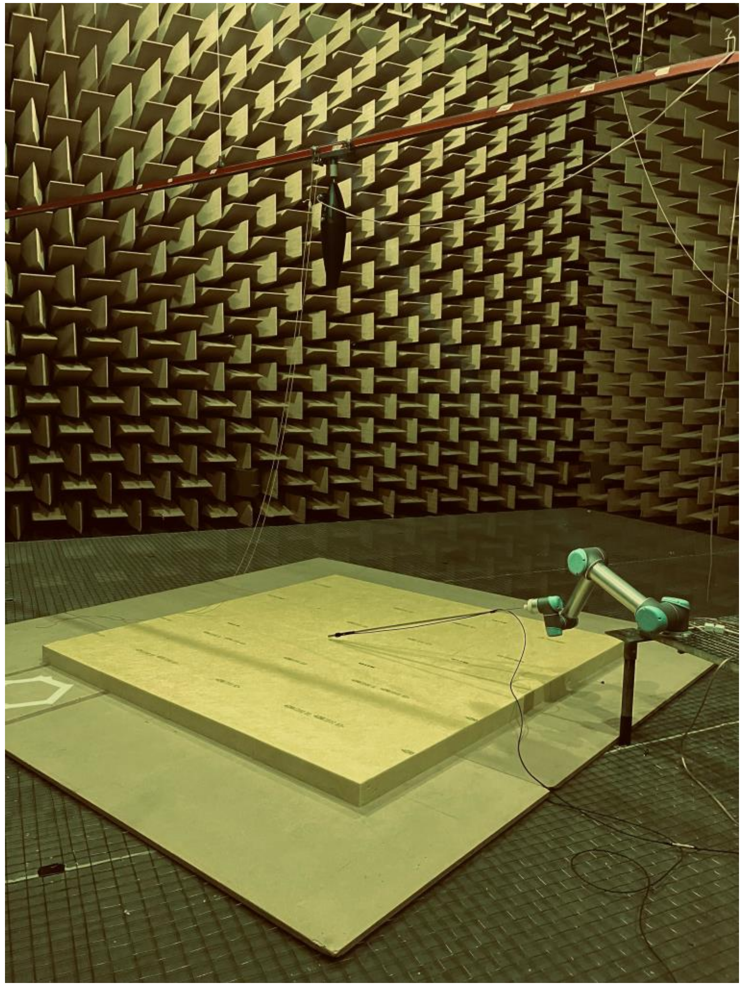
R and p_0 measured in free field



Saint-Gobain Ecophon Industry Modus 10 cm

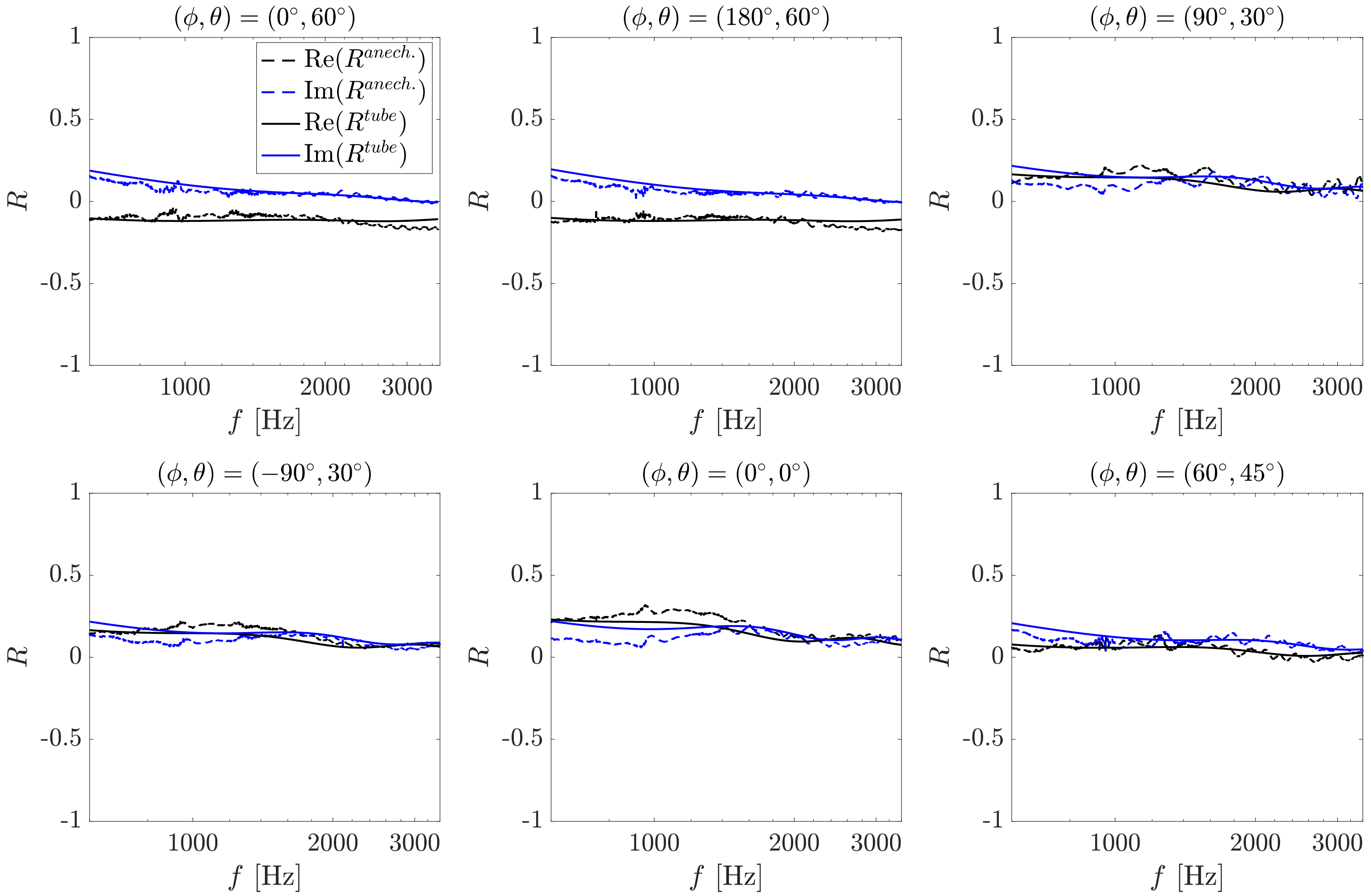
Free-field measurements

- 1000 m^3 anechoic chamber at DTU
- 2.4 m x 2.4 m layer of glass wool (10 cm Industry Modus, Saint-Gobain Ecophon) on a backing plate
- Robotic arm equipped with a microphone
- Additional microphone flush-mounted at the interface between the layer and the backing plate



Free-field measurements

- Reflection coefficients



Conclusions

- An experimental procedure has been proposed for retrieving the bulk modulus and all six components of the density tensor of a rigidly-backed layer of homogeneous anisotropic porous material
- The procedure relies on measuring the reflection coefficient in free field at various angles of incidence with an array of microphones
- Measured reflection coefficients compare reasonably well with experimental results obtained from reflection and transmission coefficients measured in an impedance tube

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Thank you!