Sorrento SAPEM' 23 常熟

Sound Absorption Coefficient of Extended-Reaction Porous Materials under Monopole Excitation: A Cautionary Examination

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UdS Université de Sherbrooke

ÉTS Le génie pour l'industrie

irsst Institut de recherche Robert-Sauvé en santé et en sécurité du travail

Motivations

- Sound Absorption Coefficient (SAC) is known to depend on the type of incident sound field. In practice, idealized incident fields such as normal, oblique incidence plane wave or diffuse field are used

Modeling framework

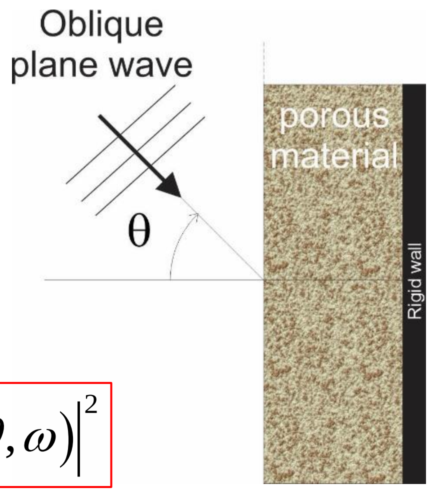
- Idealized SACs classically used as input parameters in indoor acoustic modeling
- However sound sources often represented as monopoles
- Lack of literature on SACs for finite-sized porous materials under monopole excitation

How much does the SAC of a material under monopole excitation deviate from that under idealized plane waves or diffuse fields depending on source height, material lateral extent and characteristics ?

Motivations (continued)

Measurement framework

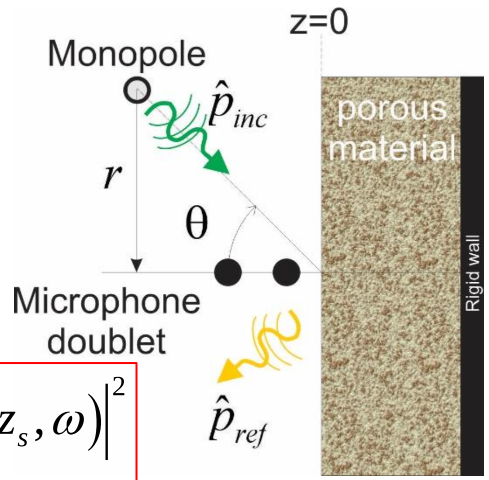
- Plane wave SAC can be derived from complex reflection coefficient



$$\hat{\Re} = \frac{\hat{p}_{ref}\left(z=0,\omega\right)}{\hat{p}_{inc}\left(z=0,\omega\right)}$$

$$\alpha_{\mathrm{pw}}\left(\theta,\omega\right) = 1 - \left|\hat{\Re}\left(\theta,\omega\right)\right|^2$$

- Approximation of $\hat{\Re}$ classically obtained using in situ methods which make use of a monopole source



$$\hat{\Re}_{\mathrm{loc}} = \frac{\hat{p}_{ref}\left(x,y,z=0,z_s,\omega\right)}{\hat{p}_{inc}\left(x,y,z=0,z_s,\omega\right)}$$

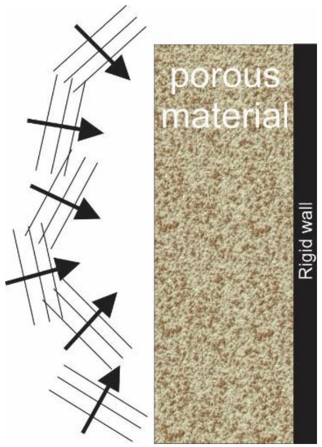
$$\alpha_{\mathrm{loc}}\left(x,y,z_s,\omega\right) = 1 - \left|\hat{\Re}_{\mathrm{loc}}\left(x,y,z_s,\omega\right)\right|^2$$

$$\alpha_{\mathrm{loc}}\left(\theta,z_s,\omega\right) = 1 - \left|\hat{\Re}_{\mathrm{loc}}\left(\theta,z_s,\omega\right)\right|^2$$

- Diffuse field SAC can be obtained by averaging over the incidence angles

Diffuse field

$$\alpha_{\mathrm{d}}\left(\omega\right) = \frac{\int_0^{\theta_{\max}} \alpha_{pw}\left(\theta\right)\cos\theta\sin\theta d\theta}{\int_0^{\theta_{\max}} \cos\theta\sin\theta d\theta}$$



Diffuse field

$$\alpha_{\mathrm{d,loc}}\left(z_s,\omega\right) = \frac{\int_0^{\theta_{\max}} \alpha_{\mathrm{loc}}\left(\theta,z_s,\omega\right)\cos\theta\sin\theta d\theta}{\int_0^{\theta_{\max}} \cos\theta\sin\theta d\theta}$$

Motivations (continued)

Measurement framework

- Approximating $\hat{\mathfrak{R}}(\theta)$ with $\hat{\mathfrak{R}}_{loc}(\theta)$ introduces some error to obtain α_{d} ➡ Uncertainty about conditions under which this error is minimized (e.g., material size, source height)

How relevant is the local plane wave assumption for assessing diffuse field SACs of finite-size absorbing materials?

How to describe the overall sound performance of the material sample by a suitable SAC when subjected to a monopole excitation and how this SAC compares to the plane wave and diffuse field SACs ?

Objectives

Main objective

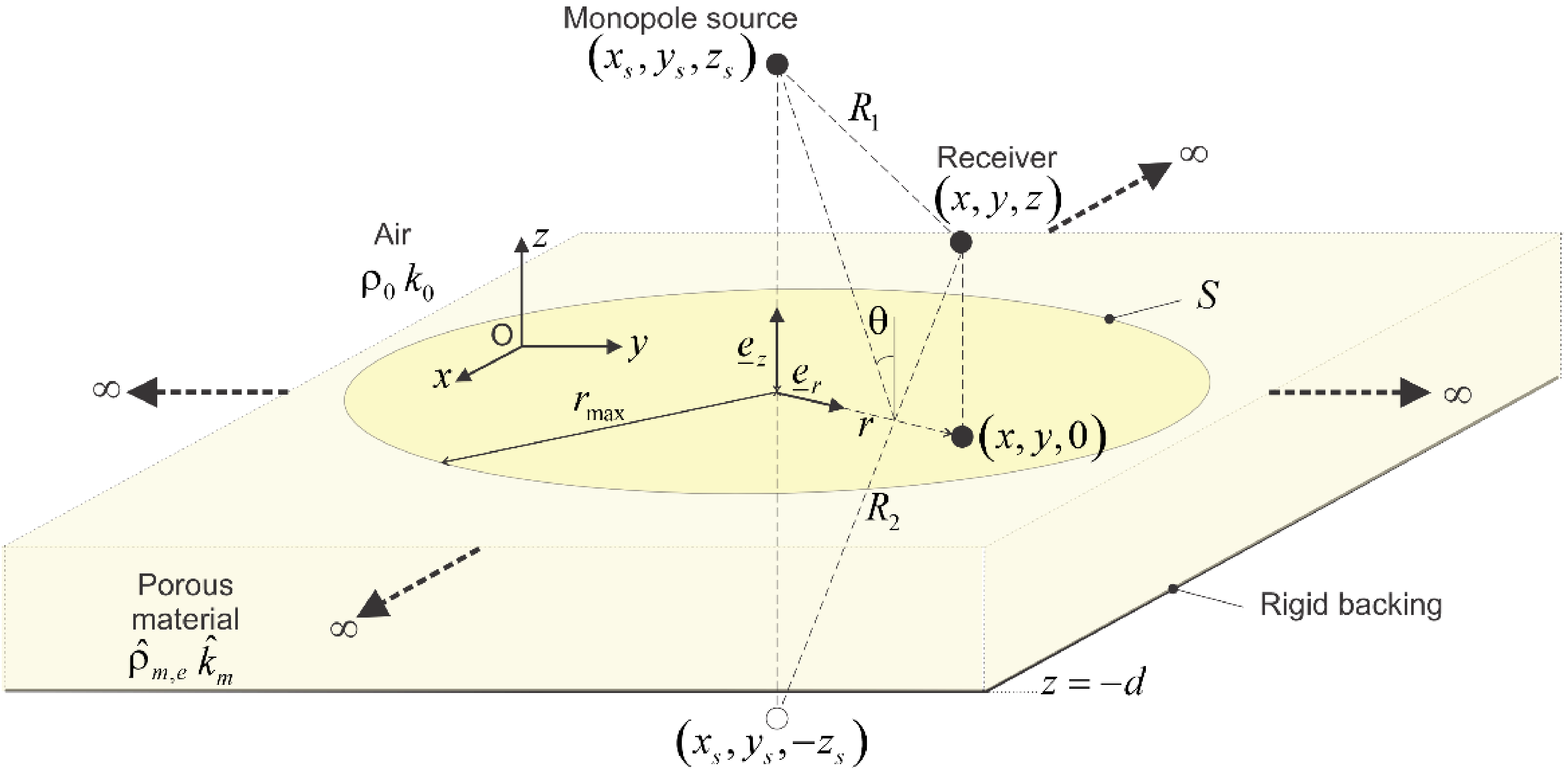
- Investigate the SAC of a finite size extended-reaction sound absorbing material when subjected to a monopole excitation based on a power ratio definition of the SAC (α_{abs}) and Allard's model [4]

Specific objectives

- Investigate how factors such as the monopole height, the material size and material characteristics (thickness) influence α_{abs}
- Investigate the extent to which the diffuse field SAC $\alpha_{\mathrm{d,loc}}$ obtained from $\hat{\Re}_{loc}(\theta)$ can approximate the traditional diffuse field SAC α_{d}
- Compare α_{abs} with $\alpha_{\mathrm{pw}}(0)$, $\alpha_{\mathrm{d,loc}}$ and α_{d}

[4] J. Allard, W. Lauriks, and C. Verhaegen, "The acoustic sound field above a porous layer and the estimation of the acoustic surface impedance from free-field measurements," The Journal of the Acoustical Society of America **91**, 3057–3060 (1992)

Theory – Allard model

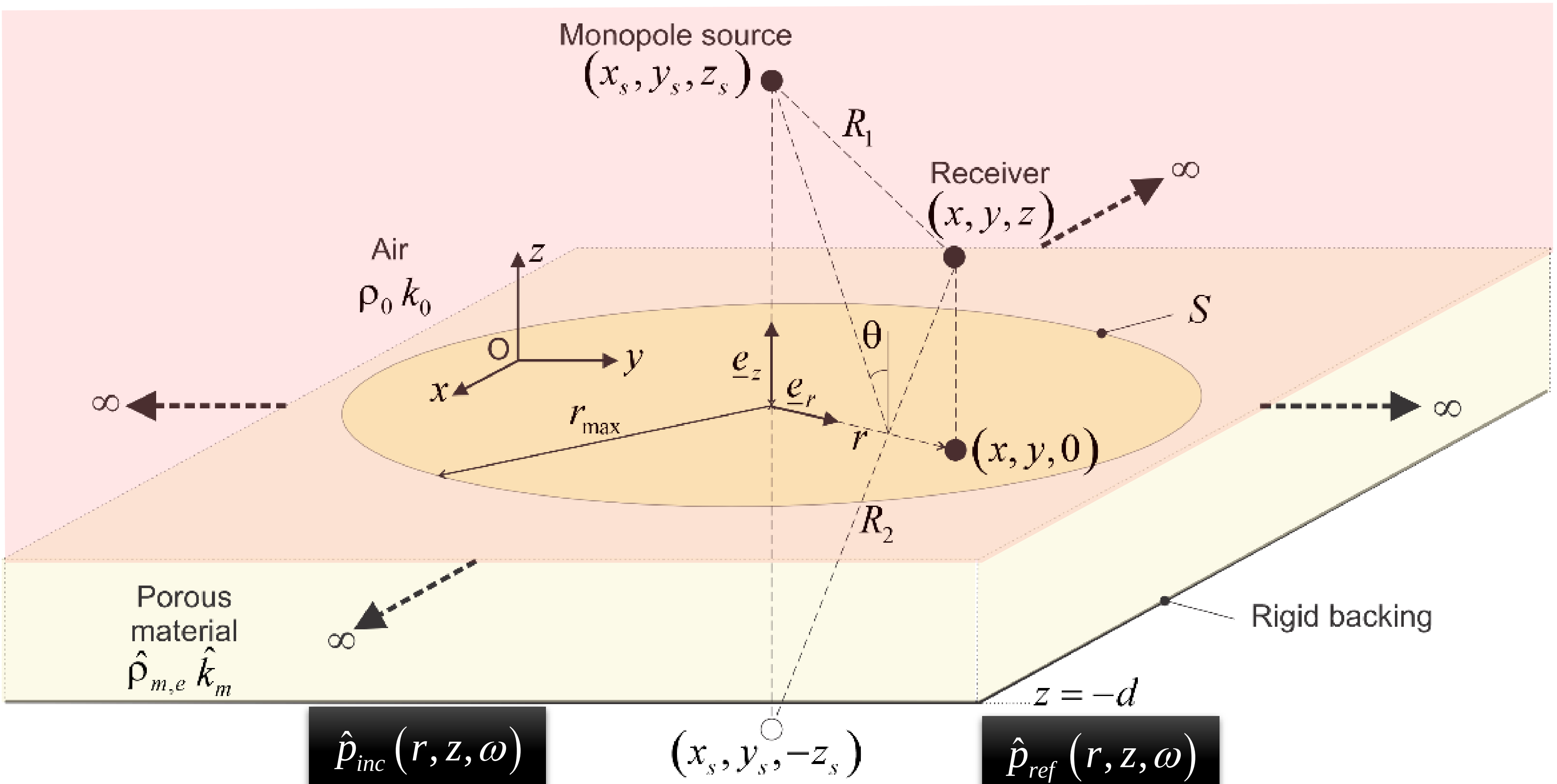


- Allows for calculating the sound field above a laterally infinite porous material excited by a monopole source [1]
- Porous material described by an equivalent fluid model

[4] J. Allard, W. Lauriks, and C. Verhaegen, “The acoustic sound field above a porous layer and the estimation of the acoustic surface impedance from free-field measurements,” The Journal of the Acoustical Society of America **91**, 3057–3060 (1992)

Theory – Allard model - field above porous material

Assume radial symmetry



$\hat{p}_{inc}(r, z, \omega)$

$\hat{p}_{ref}(r, z, \omega)$

$$r = \sqrt{(x_S - x)^2 + (y_S - y)^2}$$

$$R_1 = \sqrt{r^2 + (z_S - z)^2}$$

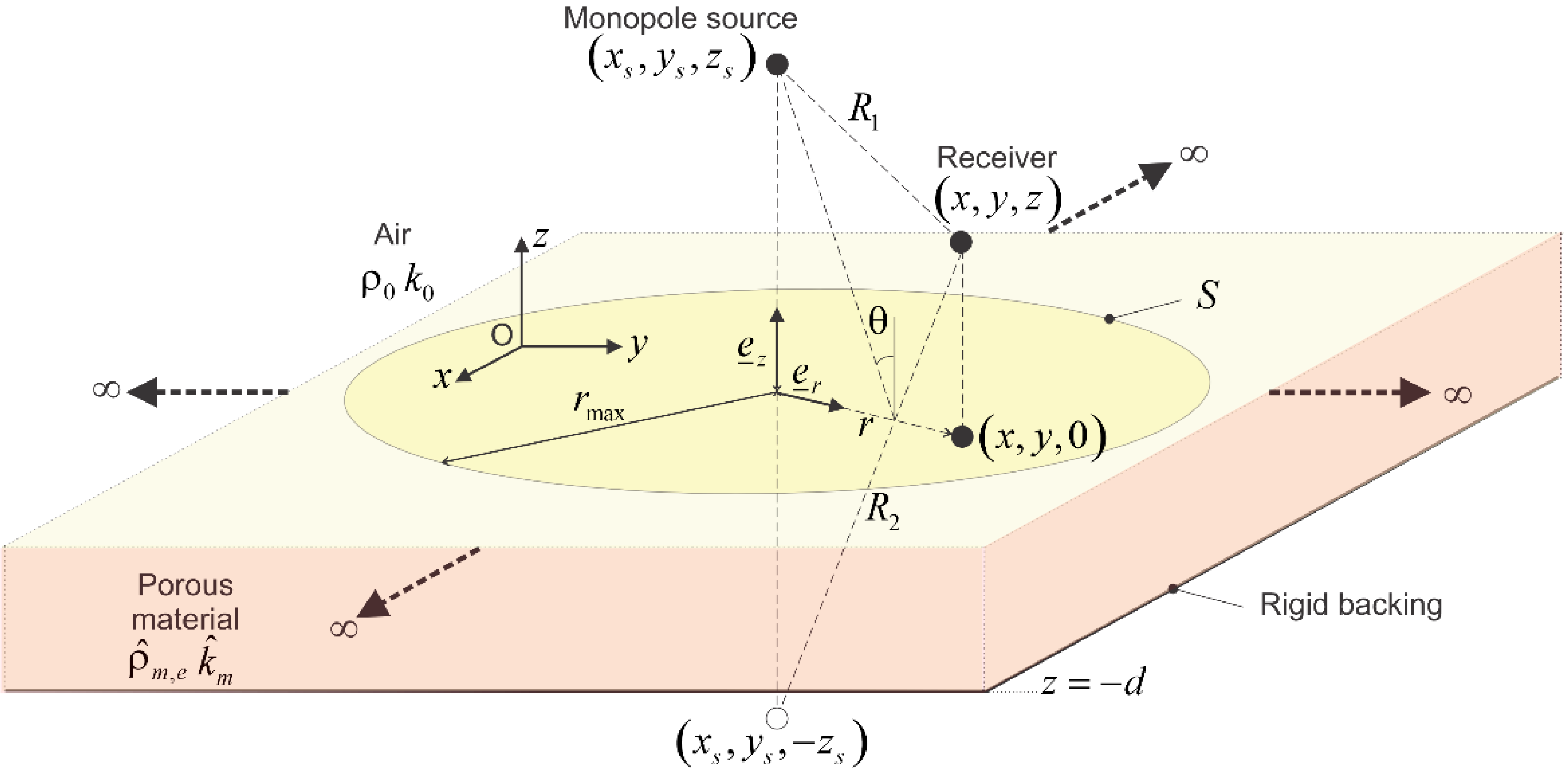
$$R_2 = \sqrt{r^2 + (z_S + z)^2}$$

$$\hat{v}_0^2 = k_r^2 - k_0^2$$

$$\hat{v}_m^2 = k_r^2 - \hat{k}_m^2$$

$$\hat{p}(r, z, \omega) = j\rho_0\omega \frac{e^{-jk_0R_1}}{R_1} + j\rho_0\omega\left(-\frac{e^{-jk_0R_2}}{R_2} + \int_0^{+\infty} \frac{2\hat{\rho}_{m,e}}{\hat{\rho}_{m,e}\hat{v}_0 + \rho_0\hat{v}_m \tanh(\hat{v}_m d)} e^{-\hat{v}_0(z_S+z)} J_0(rk_r) k_r dk_r\right)$$

Theory – Allard model – field inside porous material



- Sound field inside the porous material given by [5,6]

[5] S. Thomasson, “Sound propagation above a layer with a large refraction index,” The Journal of the Acoustical Society of America **61**, 659–674 (1977)
[6] H. Tao, B. N. Tong, and K. M. Li, “Sound penetration into a hard-backed rigid porous layer: Theory and experiments,” The Journal of the Acoustical Society of America **136**, 475–484 (2014

Theory – classical definitions of SAC

Oblique incidence
Plane wave excitation

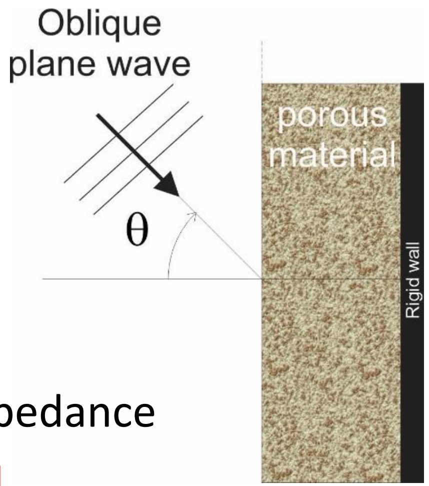
$$\hat{\Re}(\theta,\omega)=\frac{\hat{Z}_s(\theta,\omega)\cos\theta-Z_0}{\hat{Z}_s(\theta,\omega)\cos\theta+Z_0}$$

$$\hat{Z}_s=-j\hat{Z}_{c,m}\frac{\hat{k}_m\cot\left(d\sqrt{\hat{k}_m^2-k_0^2\sin^2\theta}\right)}{\phi\sqrt{\hat{k}_m^2-k_0^2\sin^2\theta}}$$

$$\hat{Z}_{c,m}=\omega\phi\frac{\hat{\rho}_{m,e}}{\hat{k}_m}$$

Z_0 : air characteric impedance

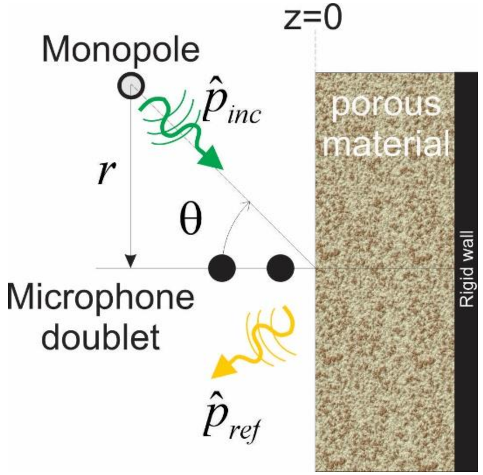
$$\alpha_{\text{pw}}(\theta,\omega)=1-\left|\hat{\Re}(\theta,\omega)\right|^2$$



Oblique incidence
Monopole excitation

$$\hat{\Re}_{loc}(\theta,\omega)=\hat{\Re}_{\text{loc}}(r,z_s,\omega)$$

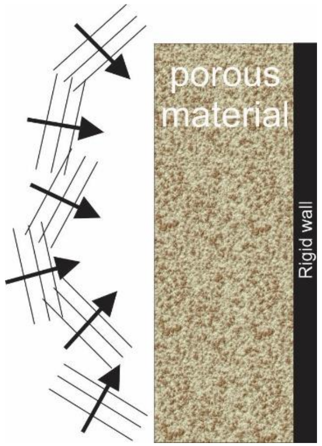
$$\hat{\Re}_{\text{loc}}=\frac{\hat{p}_{ref}(r,z=0,z_s,\omega)}{\hat{p}_{inc}(r,z=0,z_s,\omega)}$$



$$\alpha_{\text{loc}}(z_s,r,\omega)=1-\left|\hat{\Re}_{\text{loc}}(z_s,r,,\omega)\right|^2$$

Diffuse field

$$\alpha_{\text{d}}(\omega)=\frac{\int_0^{\theta_{\max}}\alpha_{pw}(\theta)\cos\theta\sin\theta d\theta}{\int_0^{\theta_{\max}}\cos\theta\sin\theta d\theta}$$

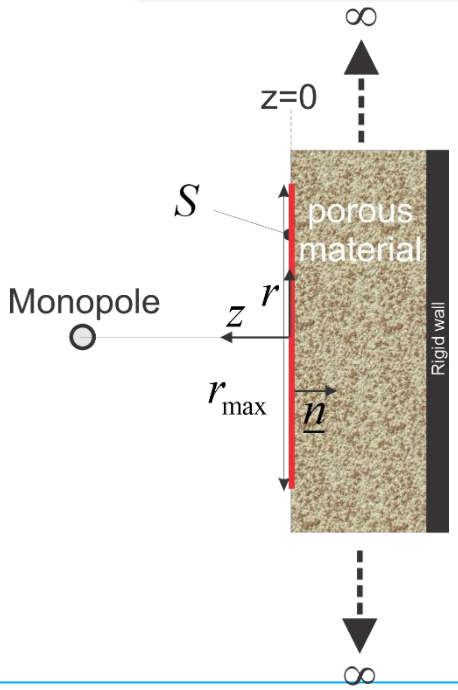


Diffuse field

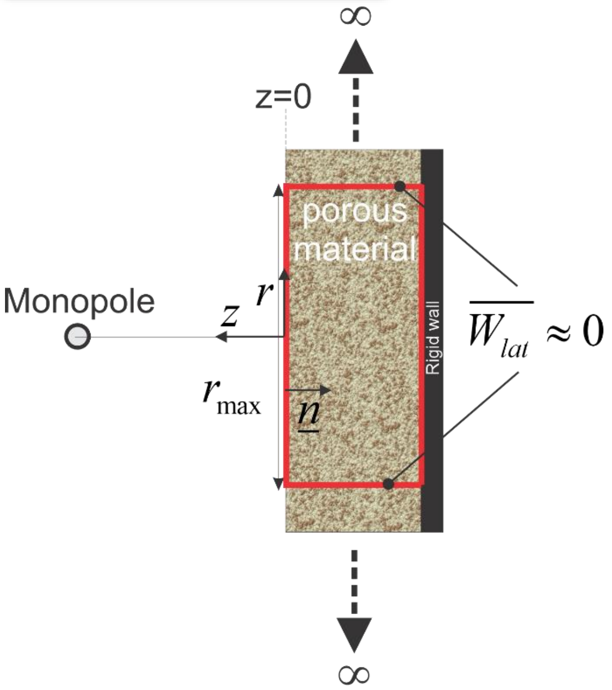
$$\alpha_{\text{d,loc}}(\omega)=2\left(\frac{z_s^2}{r_{\max}^2}+1\right)\int_0^{z_s\tan\theta_{\max}}\left(1-\left|\hat{\Re}_{\text{loc}}(r,z_s,\omega)\right|^2\right)\frac{z_s^2 r}{\left(z_s^2+r^2\right)^2}dr$$

Theory – Calculation of incident, absorbed and dissipated powers [1,2,3]

Absorbed power



Dissipated power



$$\overline{W_{inc}}(\omega) = \rho_0 \omega \pi k_0 \left[1 - \frac{z_S}{\left(r_{max}^2 + z_S^2\right)^{\frac{1}{2}}}\right]$$

$$\alpha_{abs} = \frac{\overline{W_{abs}}}{\overline{W_{inc}}}$$

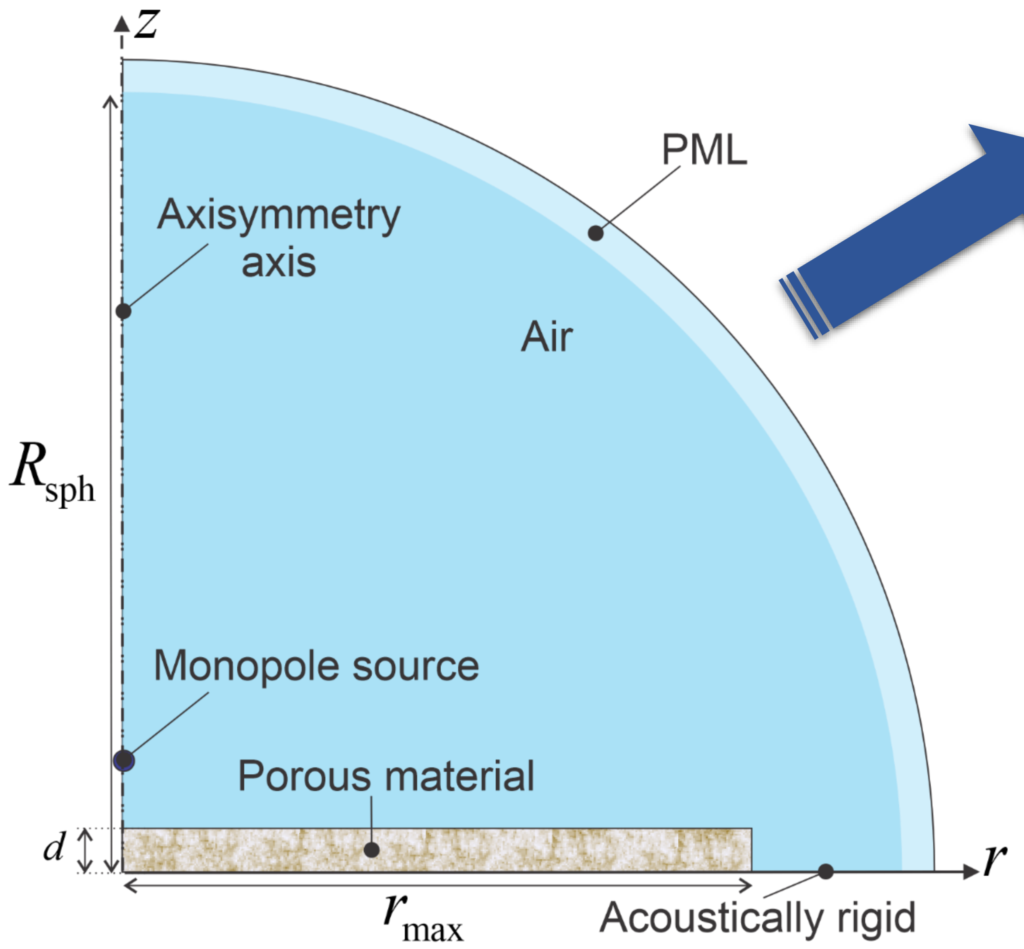
$$\alpha_{diss} = \frac{\overline{W_{diss}}}{\overline{W_{inc}}}$$

$$\overline{W_{abs}}(\omega) = 2\pi \int_0^{r_{max}} \Re\left[\left(\int_0^{+\infty} \frac{j\rho_0\omega\hat{\rho}_{m,e}}{\hat{\rho}_{m,e}\hat{\nu}_0 + \rho_0\hat{\nu}_m \tanh(\hat{\nu}_m d)} e^{-\hat{\nu}_0 z_S} J_0(rk_r) k_r dk_r\right) \times \left(2e^{jk_0 R} z_S \left(\frac{1 - jk_0 R}{R^3}\right) - \int_0^{+\infty} \frac{2\hat{\rho}^*_{m,e}\nu^*_0}{\hat{\rho}^*_{m,e}\hat{\nu}^*_0 + \rho_0\hat{\nu}^*_m \tanh^*(\hat{\nu}_m d)} e^{-\hat{\nu}^*_0 z_S} J_0(rk_r) k_r dk_r\right)\right] r dr$$

$$\overline{W_{diss}}(\omega) \approx \Im\left[-\frac{\hat{k}_m^2 \pi}{\omega\hat{\rho}_{m,e}} \int_{-d}^{0}\int_0^{r_{max}} \left|\hat{p}_m^2(r,z)\right| r dr dz\right] + \Im\left[\frac{\pi}{\omega\hat{\rho}_{m,e}} \int_{-d}^{0}\int_0^{r_{max}} \underline{\nabla}\hat{p}_m(r,z) . \underline{\nabla}\hat{p}^*_m(r,z) r dr dz\right]$$

[1] Kuipers E., Measuring sound absorption using local field assumptions, PhD thesis, U of Twente, 2013
[2] S Thomasson, “On the absorption coefficient,” Acta Acustica United with Acustica **44**, 265–273 (1980)
[3] Atalla, F. C. Sgard, R. Panneton, and X. Olny, “Acoustic absorption of macro-perforated porous materials,” Journal of Sound and Vibration **243**, 659–678 (2001)

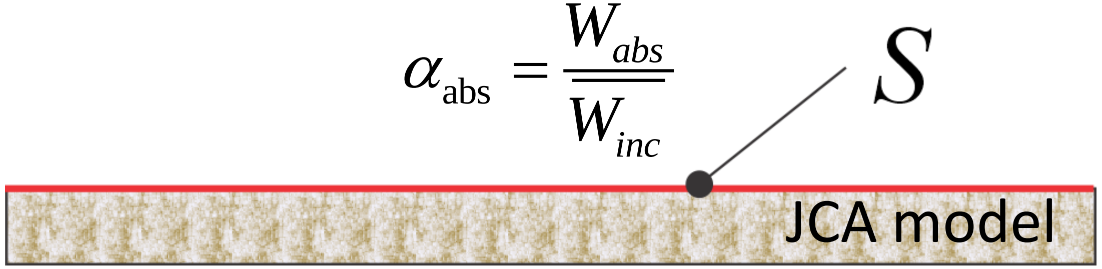
Results – Verification of the proposed approach



Finite element model (COMSOL Multiphysics)

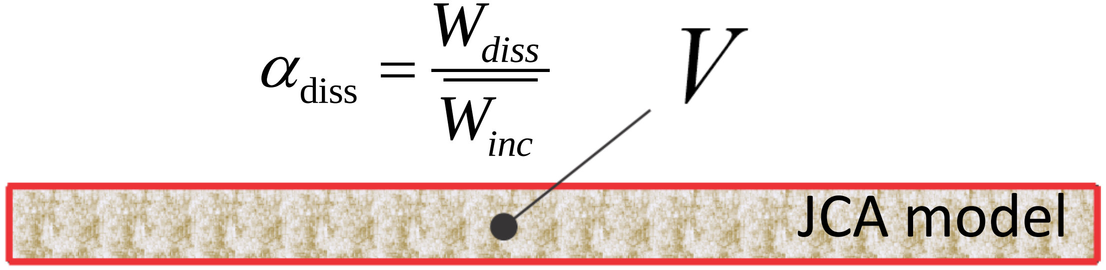
COMSOL Multiphysics + Present approach

$$\alpha_{abs} = \frac{\overline{W_{abs}}}{\overline{W_{inc}}}$$

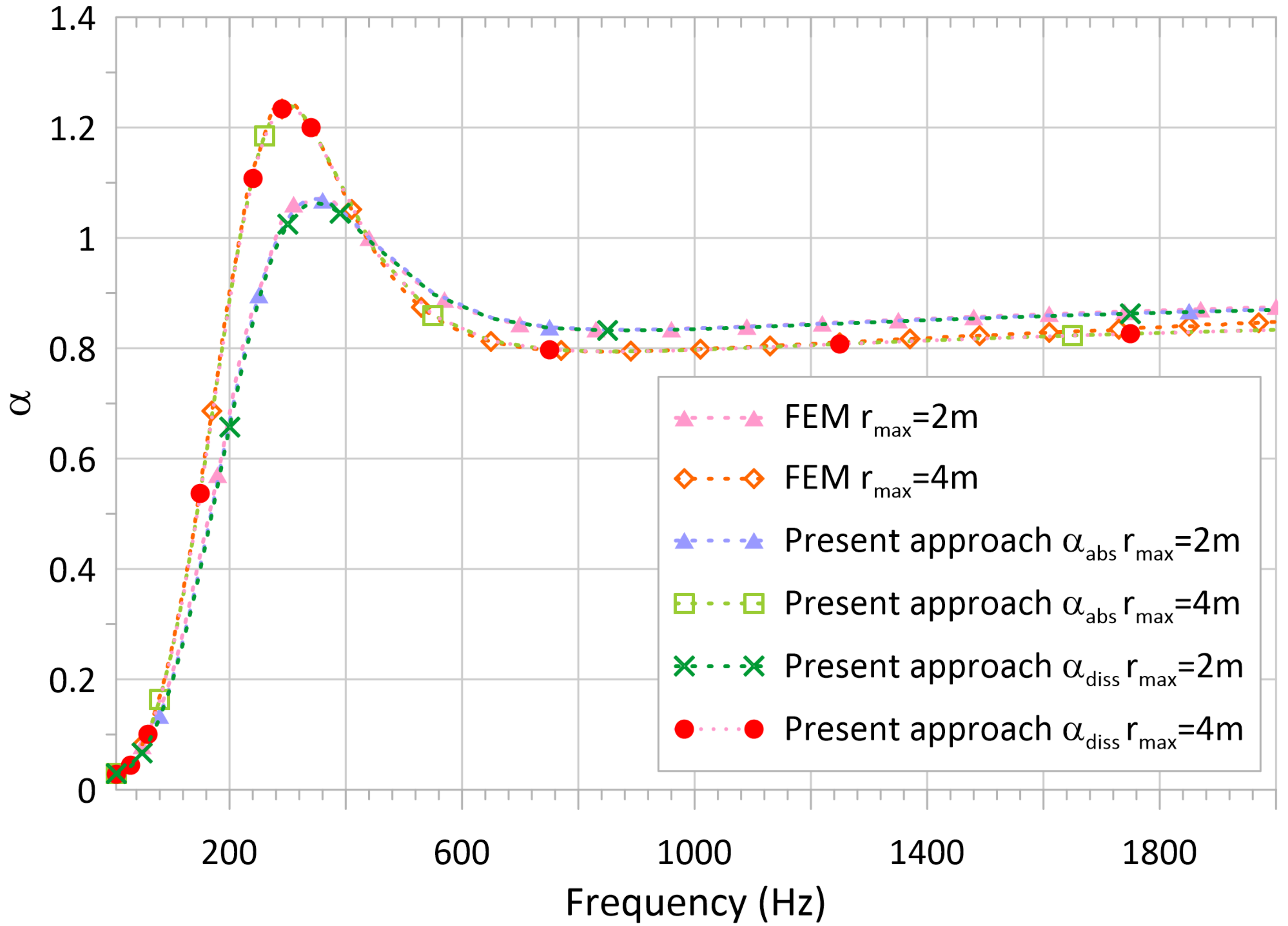


Present approach

$$\alpha_{diss} = \frac{\overline{W_{diss}}}{\overline{W_{inc}}}$$



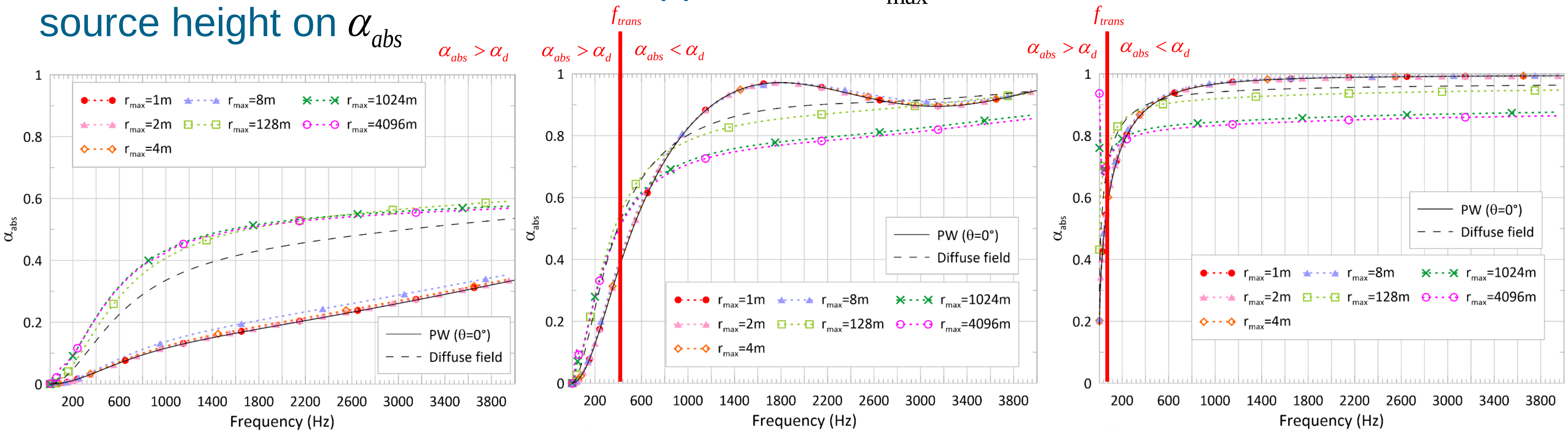
Results – Verification of the proposed approach



Melamine d=5 cm

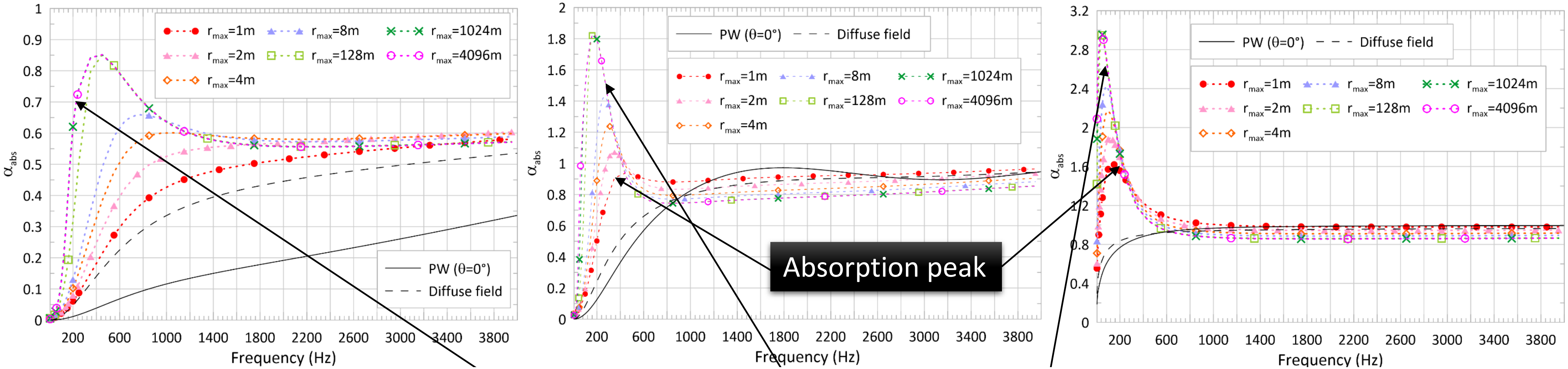
z_s=0.3 m

Results –Effect of radial distance upper bound $r_{\max}$, material thickness and source height on α_{abs}



When $r_{\max}$ is very large (infinite material), $\alpha_{abs} < \alpha_d$ for $f > f_{trans}$ & $\alpha_{abs} \geq \alpha_d$ for $f < f_{trans}$. f_{trans} ↘ as d ↗

Results –Effect of radial distance upper bound r_{max}, material thickness and source height on α_{abs}

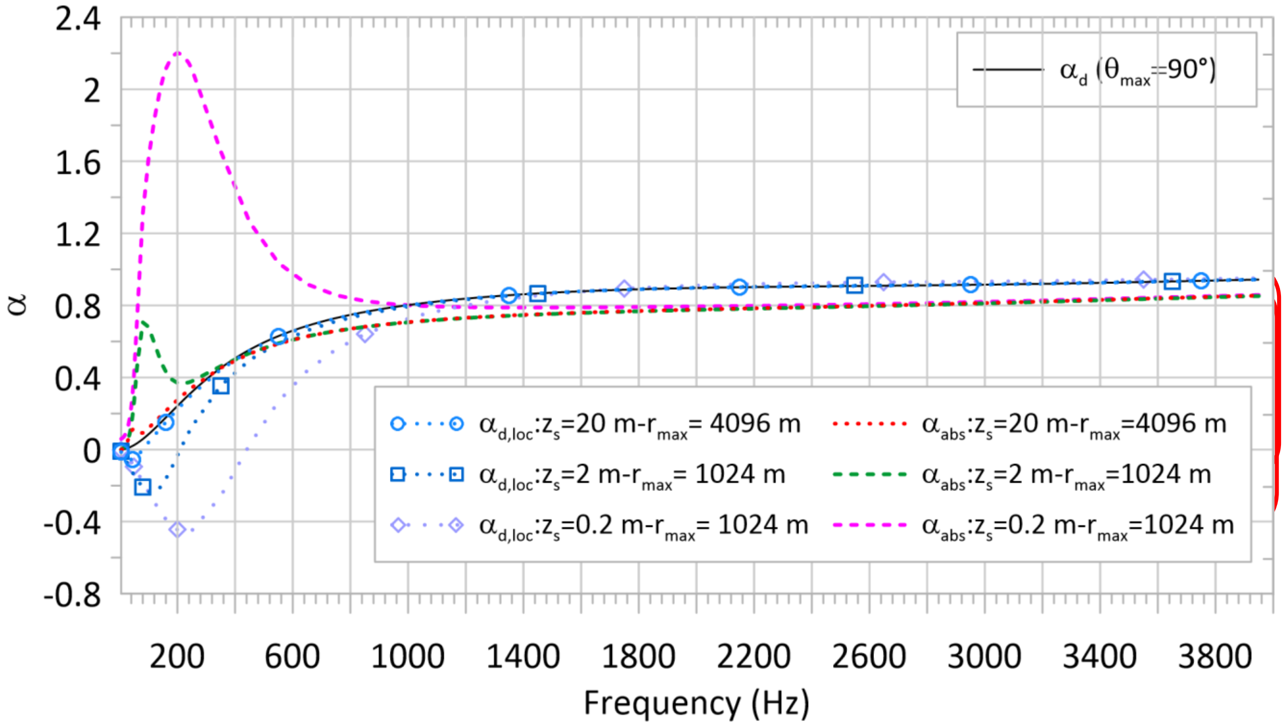


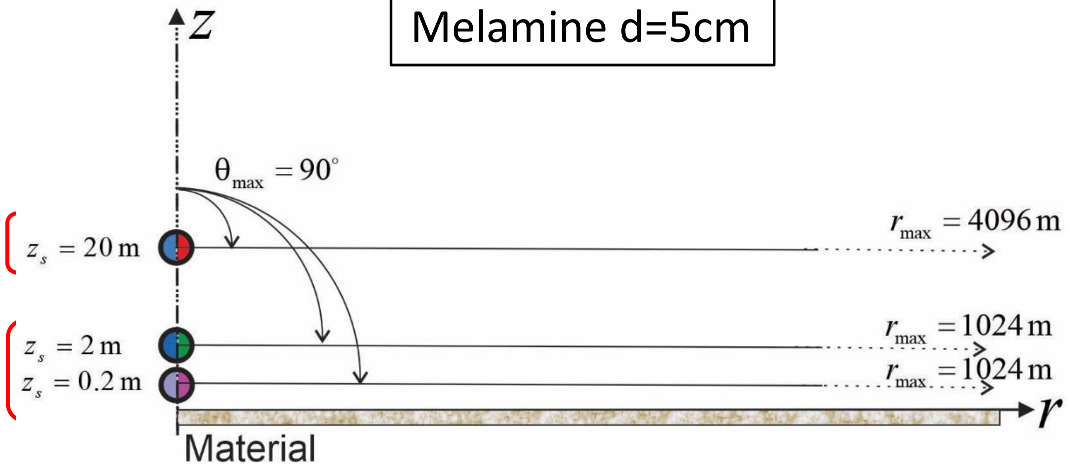
α_{abs} peak >1 can be explained by the behavior of the normal absorbed intensity field at the material surface (spatial zone where $\overline{I}_{abs} > \overline{I}_{inc}$)

Results – $\alpha_{d,loc}$ vs α_d vs α_{abs}

$$\theta_{max} = \arctan\left(\frac{r_{max}}{z_s}\right)$$

Melamine d=5cm

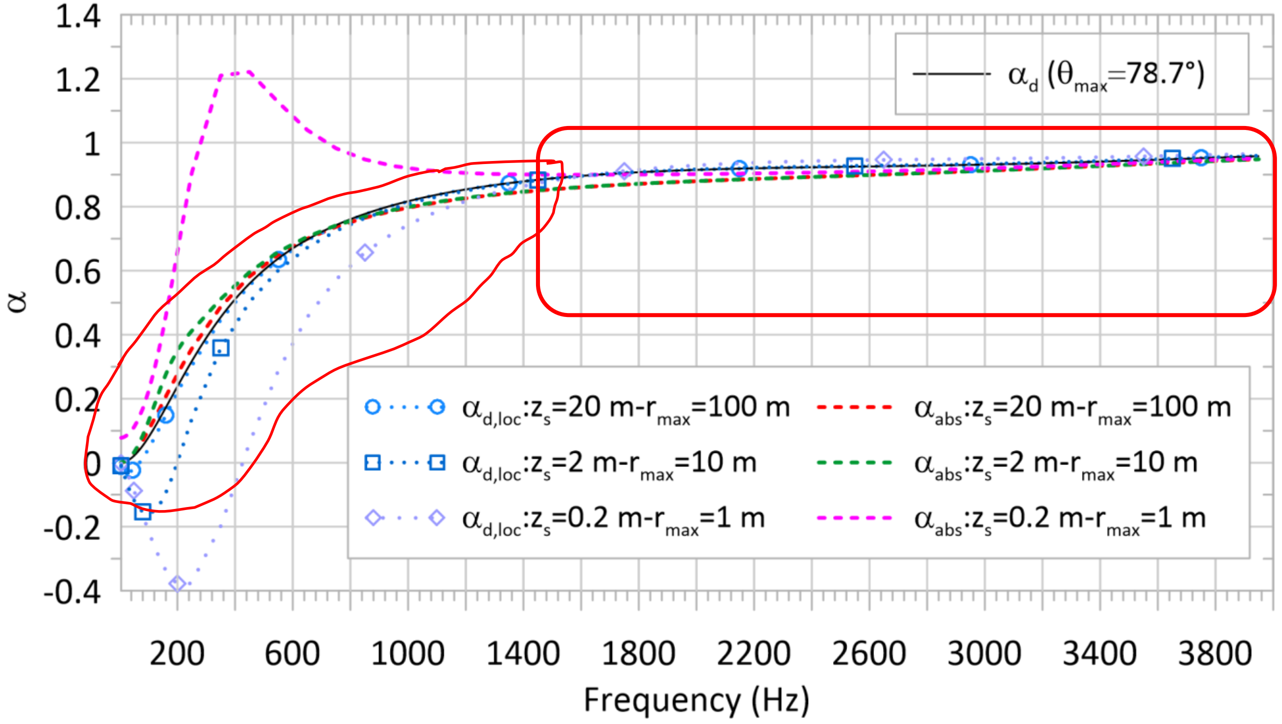




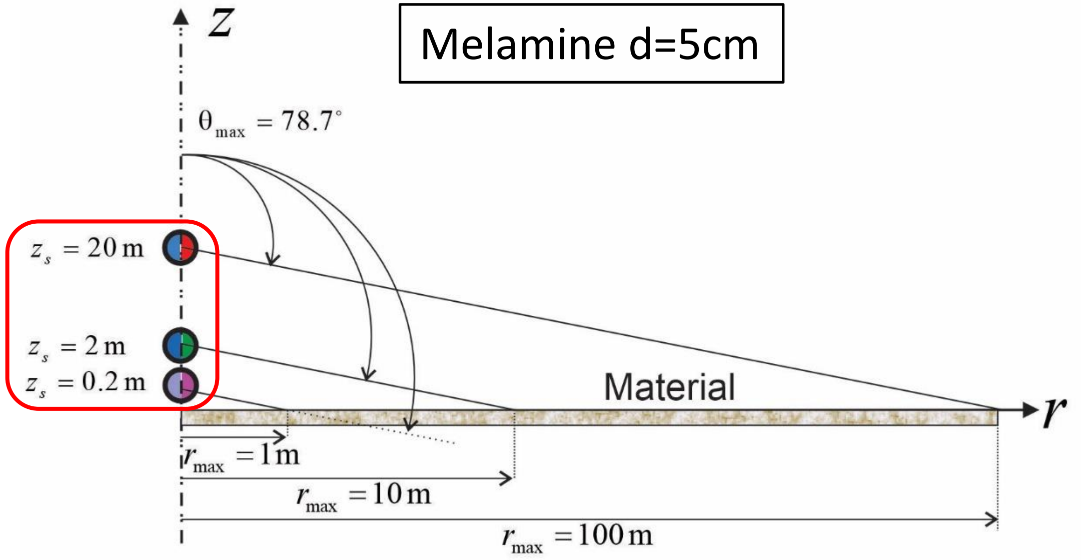
α_{abs} of an "infinite material" is different from $\alpha_{d,loc}$ and α_d especially at low frequencies and sources close to material

Results – $\alpha_{d,loc}$ vs α_d vs α_{abs}

$$\theta_{max} = \arctan\left(\frac{r_{max}}{z_s}\right)$$



Melamine d=5cm



When z_s ↗ α_{abs} gets closer to $\alpha_{d,loc}$ and α_d at low frequencies

Conclusion

- $\alpha_{abs} = \frac{\overline{W_{abs}}}{\overline{W_{inc}}}$ implicitly takes into account the material radial size r_{max} and strongly depends on it
- For certain conditions, a sound absorption peak appears at low frequencies particularly when the source is close to the material surface.
- α_{abs} >1 at low frequency for sources close to the material is due to absorbed sound intensity exceeding incident sound intensity within a substantial region not far from the source
- Converged value of α_{abs} (representing the SAC for an infinitely laterally extended material) does not align with the SAC values for normal incidence plane wave or diffuse field conditions
- For practical material sizes α_{abs} is significantly different from normal or diffuse field SAC at low frequencies when source is near the material. At medium-high frequencies, α_{abs} approaches diffuse field SAC for sufficiently absorbing materials.

This raises caution against utilizing plane wave and diffuse field SACs for characterizing the sound absorption performance of porous materials under monopole excitation in particular at low frequency.

Thank you !