Strain wave propagation through felt-like material

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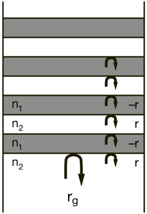
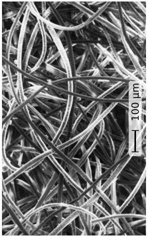
Motivation





- Metamaterials are engineered materials[1] designed to have properties that can not be found in natural materials
- They are created by arranging or structuring components at a smaller scale than the wavelength of the phenomena they interact with
- Metamaterials can be designed such that they exhibit band gaps (BG) and properties like negative group velocity (NGV)[2,3]
- These materials can be used to manipulate acoustic waves and, therefore beneficial for noise and vibration reduction and control

[1] S. Guenneau and R. V. Craster, *Fundamentals of Acoustic Metamaterials* Dordrecht: Springer Netherlands, 2013, pp. 1–42.

[2] N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, X. Zhang, "Ultrasonic metamaterials with negative modulus," *Nat. Mater.*, **5**(6), pp. 452–456, 2006.

[3] S. Guenneau, A. Movchan, G. Petursson, S. A. Ramakrishna, "Acoustic metamaterials for sound focusing and confinement," *New J. Phys.*, **9**(11), p. 399, 2007.

Main aim

- Numerically analyse the acoustic wave propagation through a felt continuum using a previously obtained visco-elastic model equation is performed
- Dispersion and dissipation analysis is presented
- Studying numerically the possible effect of NGV and BG on the wave propagation



Model equation

A 1D equation for modelling wave propagation through a felt-like medium was derived in [4]. Presented here in dimensionless variables in the following form:

$$\frac{\partial^2\left(\varepsilon^p\right)}{\partial x^2}-\frac{\partial^2 \varepsilon}{\partial t^2}+\frac{\partial^3\left(\varepsilon^p\right)}{\partial x^2 \partial t}-\delta \frac{\partial^3 \varepsilon}{\partial t^3}=0, \tag{1}$$

where $\varepsilon(x, t)$ is the strain, $p \geq 1$ is the material parameter and $0<\delta \leq 1$ is the hereditary parameter.

[4] D. Kartofelev, A. Stulov, "Wave propagation and dispersion in microstructured wool felt," *Wave Motion*, **57**, pp. 23–33, 2015.

Dispersion and dissipation analysis

Linearised form of Eq. (1) is obtained by taking $p = 1$

$$\frac{\partial^2 \varepsilon}{\partial x^2} - \frac{\partial^2 \varepsilon}{\partial t^2} + \frac{\partial^3 \varepsilon}{\partial x^2 \partial t} - \delta \frac{\partial^3 \varepsilon}{\partial t^3} = 0. \tag{2}$$

The solution to (2) can be assumed in the form

$$\varepsilon(x, t) \propto \exp(\sigma t + \mathrm{i} k x), \tag{3}$$

where σ is the attenuation, i is the imaginary unit and k is the wavenumber. Substituting (3) into (2) produces the following characteristic equation:

$$\delta \sigma^3 + \sigma^2 + k^2 \sigma + k^2 = 0. \tag{4}$$

Since k is considered to be real the σ can be written in terms of

$$\sigma(k) = \varsigma(k) + \mathrm{i}\mu(k). \tag{5}$$

Dispersion and dissipation analysis

Using (5) the assumption (3) takes the following form

$$\varepsilon(x,t) \propto \exp((\varsigma + \mathrm{i}\mu)t + \mathrm{i}kx) = \exp(\varsigma t) \cdot \exp(\mathrm{i}(\mu t + kx)). \tag{6}$$

Where ς acts in time as an attenuation coefficient and μ is characterising the dispersion. The equation (4) can be solved using (5) as a system and written as a function of wavenumber k:

$$\begin{cases} \varsigma = \dfrac{1}{12\delta S}\left[\sqrt[3]{4}S^2 - 4S + 2\sqrt[3]{2}(1-3k^2\delta)\right], \\ \mu = \dfrac{\sqrt{6}}{12\delta S}\sqrt{\sqrt[3]{2}S^4 - 4S^2(1-3k^2\delta) + 2\sqrt[3]{4}(1-3k^2\delta)^2}, \end{cases} \tag{7}$$

where

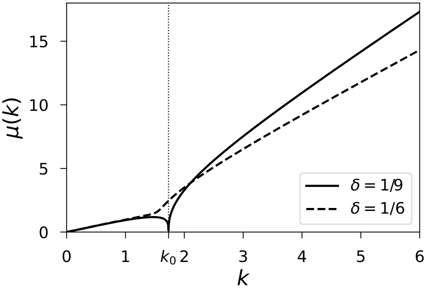
$$S = \sqrt[3]{2 - 9k^2\delta(1-3\delta) + 3k\delta\sqrt{3Q}}, \tag{8}$$

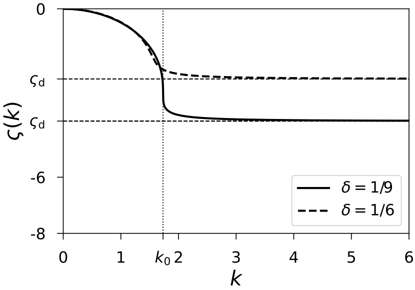
$$Q = 4k^4\delta - k^2(1 + 18\delta - 27\delta^2) + 4. \tag{9}$$

Dispersion and dissipation analysis

Three different solution regimes are distinguishable depending on the value of δ:

1. Non-dispersive and non-dissipative regime, for $\delta = 1$
2. **Low hereditary** regime with a continuous dispersion and dissipation curves, for $1/9 \leqslant \delta < 1$

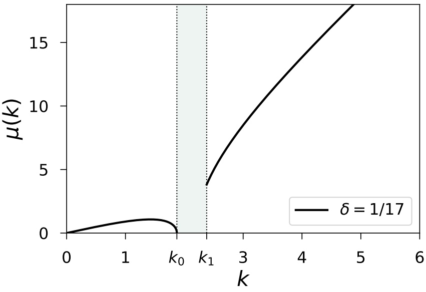


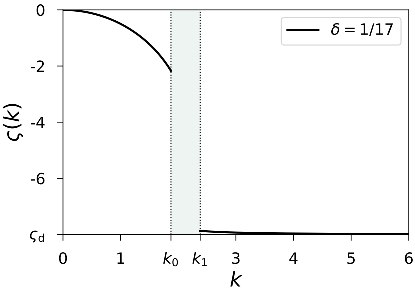


Dispersion and dissipation analysis

Three different solution regimes are distinguishable depending on the value of δ:

3. **High hereditary** regime, when $0 < \delta < 1/9$ dispersion and dissipation graphs have a BG region





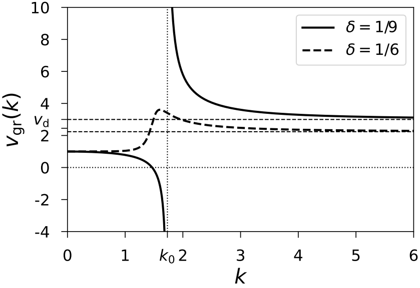
At the limit $\delta \to 0$ the BG widens since $k_0 \to 2$ and $k_1 \to \infty$.

Negative group velocity

The group velocity is given by

$$v_{\mathrm{gr}}(k) = \frac{\mathrm{d}\mu}{\mathrm{d}k}. \qquad (10)$$

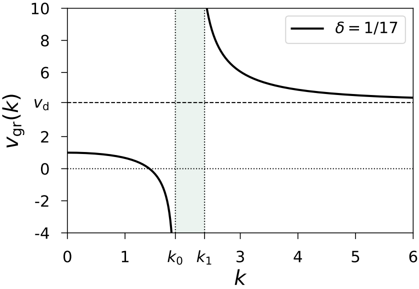
A region of NGV appears for $0 < \delta \lesssim 0.134$.



Here $\delta = 1/9 \approx 0.111$, and $\delta = 1/6 \approx 0.167$

Negative group velocity

For $0 < \delta < 1/9$ the NGV region is neighboured by the BG.



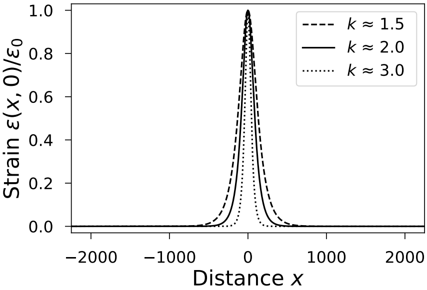
The initial value problem (IVP), $\delta = 1/17$

$$\frac{\partial^2 \varepsilon}{\partial x^2} - \frac{\partial^2 \varepsilon}{\partial t^2} + \frac{\partial^3 \varepsilon}{\partial x^2 \partial t} - \delta \frac{\partial^3 \varepsilon}{\partial t^3} = 0$$

A bell-shaped initial condition (IC) is selected in the following form:

$$\varepsilon(x, 0) = \varepsilon_0 \operatorname{sech}(\beta x), \tag{11}$$

where the characteristic width and amplitude are β and ε_0, respectively.



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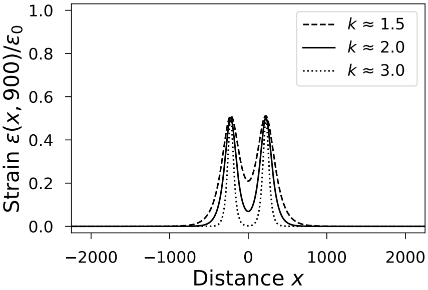

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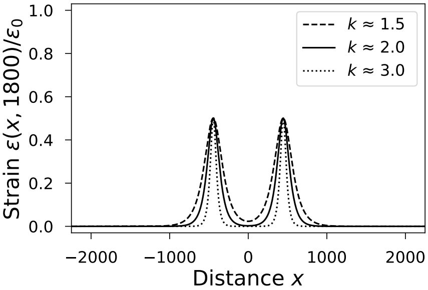

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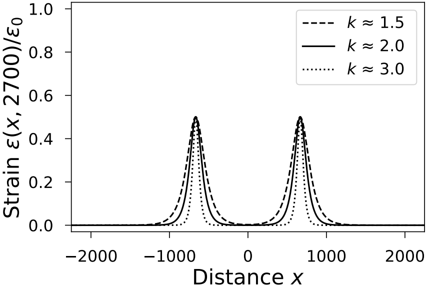

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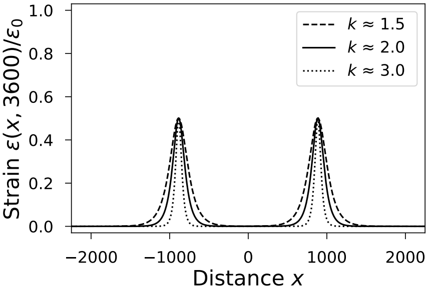

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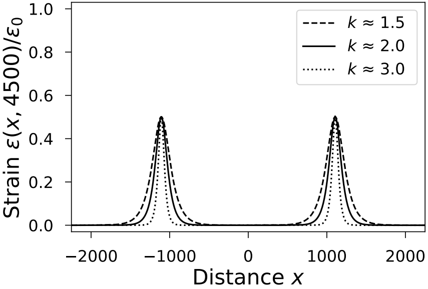

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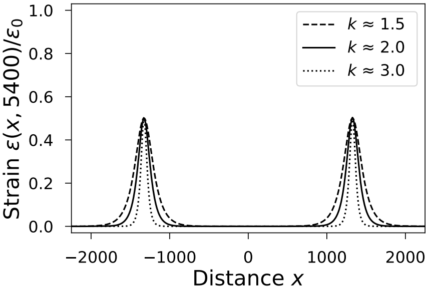

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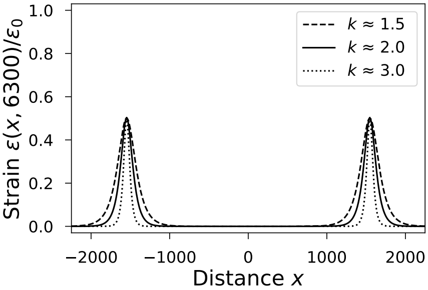

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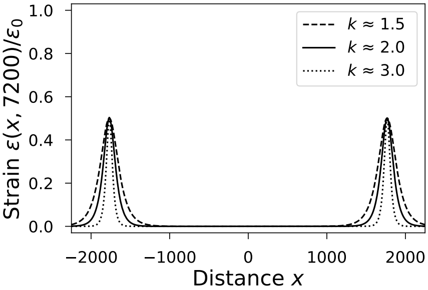

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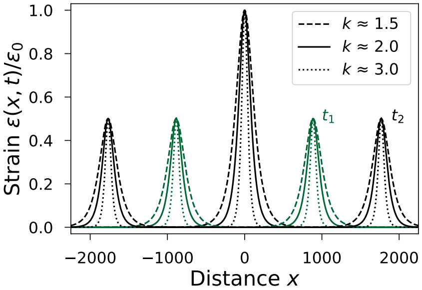


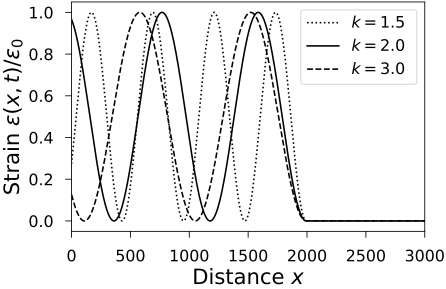
Boundary value problem (BVP), $\delta = 1/17$

$$\frac{\partial^2 \varepsilon}{\partial x^2} - \frac{\partial^2 \varepsilon}{\partial t^2} + \frac{\partial^3 \varepsilon}{\partial x^2 \partial t} - \delta \frac{\partial^3 \varepsilon}{\partial t^3} = 0$$

A sinusoidal boundary condition (BC) in the form

$$\varepsilon(0, t) = \frac{\varepsilon_0}{2} \left[1 - \cos(2\pi\gamma t)\right], \tag{12}$$

where ε_0 is the initial amplitude of the signal, γ is the signal frequency.



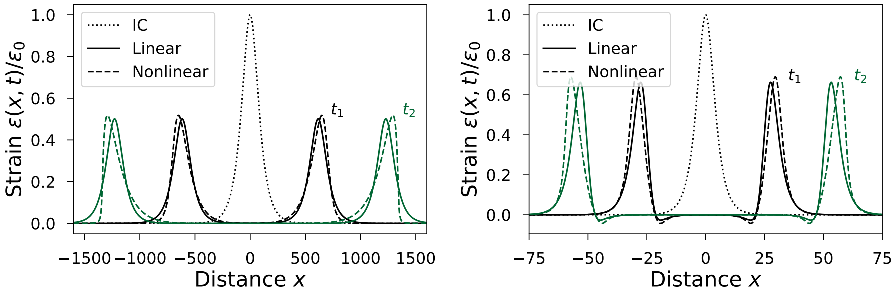
Linear vs. weakly nonlinear, $\delta = 1/17$

$$\frac{\partial^2 (\varepsilon^p)}{\partial x^2} - \frac{\partial^2 \varepsilon}{\partial t^2} + \frac{\partial^3 (\varepsilon^p)}{\partial x^2 \partial t} - \delta \frac{\partial^3 \varepsilon}{\partial t^3} = 0$$

The IC is selected in the following form:

$$\varepsilon(x,0) = \varepsilon_0 \operatorname{sech}(\beta x).$$

A linear case where $p = 1$ and a weakly nonlinear case where $p = 1.1$ for $k \approx 2$ (left) and $k \approx 50$ (right).



Conclusions

- Model equation for modelling strain wave propagation through felt was analysed
- Three solution regimes were identified depending on the values of δ
- Dispersion and dissipation analysis of the model was presented
- The BG and NGV effects on pulse evolution were studied using pulses with characteristic frequencies that corresponded to spectral wave components (k) located in BG and NGV regions
- The BG and NGV regions did not have a noticeable distorting effect on wave evolution because the characteristic wavelengths of waves were relatively large compared to the microstructure found in the felt

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